F. A. ADAMS'

COMMON
AND
HIGH SCHOOL
ARITHMETIC.

Fairmount Water-Works, Philadelphia.

PUBLISHED BY
THOMAS, COWPERTHWAIT & CO.,
PHILADELPHIA.

REVISED AND IMPROVED EDITION.

ARITHMETIC:

IN TWO PARTS.

PART FIRST,

ADVANCED LESSONS IN MENTAL ARITHMETIC.

PART SECOND,

RULES AND EXAMPLES FOR PRACTICE IN WRITTEN ARITHMETIC.

FOR COMMON AND HIGH SCHOOLS.

BY FREDERIC A. ADAMS, A.M.,
FORMER PRINCIPAL OF DUMMER ACADEMY.

THIRTEENTH THOUSAND.

PHILADELPHIA:
THOMAS, COWPERTHWAIT & CO.

New York, Roe Lockwood & Son:—Boston, Phillips, Sampson & Co; B. B. Mussey & Co.; W. J. Reynolds & Co.:—Baltimore, Cushing & Bailey:—Charleston, S. C., McCarter & Allen:—Louisville, Ky., Morton & Griswold; Beckwith & Morton:—St. Louis, Fisher & Bennett; Wm. D. Skillman; Amos H. Shultz:—Cincinnati, J. F. Desilver:—Nashville, Wm. T. Berry; Chas. W. Smith:-Memphis, C. C. Cleaves:-Lexington, C. S. Bodley:—Macon, Geo., J. M. Boardman:—Buffalo, T. & M. Butler.

1852.

RECOMMENDATIONS.

From Mr. George B. Emerson, Boston.

I have carefully examined the plan of Mr. Adams's work on Mental Arithmetic, and have given some attention to its execution; and I am confident that it will prove a very valuable addition to the means of instruction in Arithmetic. It is a successful extension of the admirable method of Colburn's First Lessons, with such modifications as seemed to be required in a higher work on the same general model. It occupies unappropriated ground; and it deserves, and I think it will take, a high place amongst the text-books. GEO. B. EMERSON.

From Mr. Thomas Sherwin, Boston.

I have carefully examined, in manuscript, the work of Mr. Adams on Mental Arithmetic, and am much pleased with it. His plan is good, and well executed. I would, therefore, heartily recommend his book to Teachers and School Committees, as one which will contribute very materially to the attainment of that very important, but much-neglected branch of study, Intellectual Arithmetic.

T. SHERWIN, *Principal of Boston English High School.*

From Professor Chase, of Dartmouth College.

MR. F. A. ADAMS. HANOVER, OCT. 12, 1846.

My Dear Sir: — I have examined, with some care, your treatise on Arithmetic, and am much pleased with it. The practice and habit of extending mental operations to large numbers is of great utility. I have occasion, very frequently, to see the inconvenience that young men suffer from the want of such a habit. Not less valuable than the habit of operating mentally upon large numbers, is the habit of performing the more advanced operations of arithmetic without the aid of the pencil.

I like very much, also, the manner in which you have treated several of the principles which you have developed; as, for example, the subject of the common divisor, the least common multiple, the roots, ratio, and proportion. These are but few of the subjects, but I mention them as examples.

I think the book will do much to promote the proper method of teaching arithmetic, — by DEMONSTRATION and explanation. I am, Dear Sir, very truly yours, &c.

S. CHASE.

From Mr. John Tatlock, Professor of Mathematics, and Mr. A. Hopkins, Professor of Natural Philosophy.

WILLIAMS COLLEGE, NOV. 20, 1846.

I have examined a treatise on Arithmetic by F. A. Adams, and am much pleased with it. I think it well adapted to teach the science and art of numbers, and at the same time to teach the art of thinking. I am persuaded that a thorough training in this Arithmetic would prepare students for the farther study of mathematics better than nine tenths are now prepared.

I should be glad if every student who enters college was master of this Arithmetic.

JOHN TATLOCK.
A. HOPKINS.

IN SCHOOL COMMITTEE, ROXBURY, FEB. 17, 1847.

The undersigned, members of the Committee on Text-Books, recommend that the Arithmetic prepared by Frederic A. Adams, be used in the grammar schools, by all the classes which now use Leonard's Arithmetic; and that hereafter no other Arithmetic be used in the public schools, than Adams's together with Colburn's First Lessons.

SAM'L. H. WALLEY, Jr. DAN'L. LEACH,
B. E. COTTING, THEO. PARKER.
GEO. PUTNAM,

In School Committee, Feb. 17, 1847. — The above recommendation of the Committee on Text-Books, was this day adopted. JOSHUA SEAVER, Secr'y.

STEREOTYPED AT THE BOSTON TYPE AND STEREOTYPE FOUNDRY.

PREFACE.

THE study of Arithmetic in the schools of this country received its best impulse, unquestionably, in the publication of Colburn's First Lessons. The use of this book, and of others made on a similar plan, has done much towards placing this branch of study on its proper ground. In all our best schools, in the elementary stages of this study, the *logical mode* has taken the place of the merely *formal*; *reason* is the guide, instead of *rule*.

The want of a higher work on Mental Arithmetic has long been felt by teachers; and by business men, who have been compelled, after having received all the training of the school-room, to adopt practical modes of calculation of their own, to meet the exigencies of their daily business. It is to meet these wants that the following work has been prepared. The design of it is,—

To accustom the pupil to perform, with ease and readiness, mental calculations upon somewhat large numbers;—

To present these operations in their natural form, freed from the inverted and mechanical methods which belong of necessity to operations in Written Arithmetic;—

To train the student to such a power of apprehending the relations of numbers, as shall give him an insight into the grounds of the rules of Arithmetic; and, consequently, shall release him from dependence on those rules;—

To prepare the members of our schools, when they shall have left school, and engaged in the active pursuits of life, to solve mentally, and with ease and delight, a large share of those questions, of busi-

ness or curiosity, for which a process of ciphering is usually thought indispensable.

The "Advanced Lessons" presuppose, of course, the knowledge of some more elementary book. To study this work successfully, therefore, the pupil must be acquainted with Colburn's First Lessons, or some other work occupying essentially the same ground.

In all the mental calculations in large sums, it will be found a uniform characteristic of this work, to begin with the highest order of numbers in the sum, — hundreds before tens, tens before units. In this way, the numbers are presented in the same order in which they are presented in the common usage of our language. In most of the operations of Written Arithmetic, however, the smallest number is taken first; and thus a method is pursued, the reverse of what the genius of our language would naturally suggest. Another advantage of taking the highest numbers first, in Mental Arithmetic, is, that we thus obtain a large approximation to the final answer, at the first step. When the first step, however, as in written addition or multiplication, furnishes only the units of the answer, leaving the hundreds or thousands still unknown, only a minute fraction of the answer is at first obtained. It is too plain to require proof, that that method will be most interesting and gratifying to the mind, which secures the largest portion of the answer at the first step. Another advantage of the method here used, is found in the fact, that we naturally make the higher order the standard; and the lower order takes its value in the mind from a comparison with the higher, as a certain part of it. Thus 150 is apprehended by the mind, as one hundred and half a hundred. This is not, indeed, the method of *acquiring* the idea of large numbers, but the method of combining them after the idea has been acquired; consequently, it is the legitimate method of instruction, just as soon as the pupil is qualified to enter on the study of such combinations. If, now, we obtain the number of the highest order first, we have a standard, under which all the succeeding orders naturally fall, and from a comparison with which they successively take their value. If we begin with units, however, and work upward through the higher

orders, we obtain no standard; we must hold the successive numbers in suspense, until the last term shall furnish the nucleus for the group, — the standard under which all the lower orders shall take their rank.

It is on the basis of these facts, which are only indications of the laws of the mind, that, throughout the mental part of this Arithmetic, the author has, in all operations, taken the highest order of numbers first. The increased interest which the persevering use of this method will awaken in the minds of pupils, will be, to teachers, a better commendation of its correctness, than any more extended mental analysis.

Other characteristic features of the Advanced Lessons are, the extended Multiplication Table; the use of the complement in addition; the analytical treatment of fractions, vulgar and decimal; the careful separation of the three topics, linear, square, and solid measure; the construction of the square and the cube; and the mode of treating proportion.

The Second Part contains examples in Written Arithmetic on all the most important rules. They are designed to be sufficiently numerous to lead the student to ready and accurate practice in ciphering. In this Part the author has aimed to interest the scholar by furnishing him with natural and reasonable questions, and to aid both teacher and scholar by arranging them progressively.

The rules and explanations will, probably, be found sufficient, after a thorough mastery of the First Part. It is not necessary that the pupil complete the First Part before beginning the Second. He may carry on both Parts at the same time; but, under each particular head, the mental part should be thoroughly mastered before the written examples are begun.

The answers to the questions in the Second Part are given in a separate work. This course has seemed to the author, on the whole, the best, notwithstanding some incidental disadvantages that may arise from it. It will enable the teacher to oversee a much larger amount of work in Arithmetic than he could otherwise attend to.

The Key will be bound up with the Arithmetic, for the use of teachers; and such copies will be lettered "*Teacher's Copy.*"

The present contains a considerable number of examples more than the Third Edition, but no change in the numbering of the sections or of the examples, to occasion inconvenience to the teacher.

To aid in awakening a higher interest and zeal in this branch of study, the author will offer a few suggestions.

Let the Key be used as little as the teacher's necessities will permit.

Let original questions be proposed by the teacher in connection with every Section.

Each member of the class should be encouraged to propose original questions to be solved by the class.

It will often be useful, especially in a review, to alter some one figure in the conditions of each question. This often produces a happy excitement, and gives quite a new zest to the study.

REVISED EDITION.

A RECENT revision of the work has led to a few alterations in the First Part, designed to render the pupil's course more strictly progressive. A thorough acquaintance with the author's First Book in Arithmetic, embracing the "lessons for practice in rapid calculation," will prepare the pupil to enter successfully on the study of this book. It is indispensable, however, to the rapid progress of the pupils, that they commence at the beginning, making the thorough mastery of each successive lesson the basis of the studies that follow.

ORANGE, New Jersey,
September 1, 1850.

CONTENTS.

PART FIRST.

PART SECOND.

EXPLANATIONS.

1 The sign = indicates equality; as, 7 times $3=21$.

2. The sign + indicates addition; as, $15+7=22$.

3. The sign —, placed between two numbers, indicates that the latter number is to be taken from the former; as, $9-4=5$.

The larger number is called the ***minuend;*** the smaller, the ***subtrahend.***

4. The sign × indicates multiplication; as, $6\times7=42$.

The two numbers are called ***factors;*** the number multiplied is called the ***multiplicand;*** the number by which it is multiplied, the ***multiplier.***

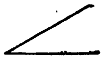
5. The sign ÷ indicates that the number placed before it is to be divided by the number after it; as, $15\div5=3$.

The number to be divided is called the ***dividend;*** the number by which it is divided is called the ***divisor.***

6. When a number is multiplied by itself, the product is called the ***second power*** of that number, or the ***square*** of it; as, $2\times2=4$, which is the second power, or the square of 2; so 9 is the square of 3; 25, the square of 5.

7. When a number is multiplied by itself, so as to be taken 3 times as a factor, the product is called the ***third power*** or the ***cube*** of the number; thus, 8 is the cube of 2, for it is formed by multiplying $2\times2\times2$; 27, or $3\times3\times3$, is the cube or third power of 3; 125, or $5\times5\times5$, is the third power of 5. The number thus used as a factor, is called the ***root*** of the power; thus, 3 is the square root of 9, and the cube root of 27; 5 is the square root of 25.

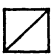
The number of the power may be expressed by a small figure; thus, 2^3 is the 3d power of 2; 3^2 is the 2d power of 3; 5^3 is the 3d power of 5.



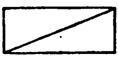
An *angle* is formed when two lines meet, running in different directions.

A *triangle* is a figure bounded by three straight lines. It is called a triangle, because it has three angles. An equilateral triangle has all its sides equal.

A *right angle* is formed when one line meets another, making the angles on both sides equal.

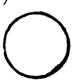


A *square* is a four-sided figure, the sides of which are all equal, and the angles of which are right angles. The diagonal divides it into two equal parts.



A *rectangle* is a four-sided figure, the opposite sides of which are equal, and the angles of which are right angles. The diagonal divides it into two equal parts.

A *parallelogram* is a four-sided figure, the opposite sides of which are equal and parallel. The diagonal divides it into two equal parts.



A *circle* is a figure bounded by a curved line, called the *circumference*, every part of which is equally distant from the centre.

A straight line from the centre to the circumference is called the *radius*.

The *diameter* is a line drawn from side to side of the circle, through the centre. It follows that the diameter is equal to twice the radius.

Any portion of the circumference, considered by itself, is called an *arc*.

A *sector of a circle* is a portion of it bounded by two radii and the arc between them.

A *sphere* is a solid bounded by a curved surface every part of which is equally distant from the centre of the solid.

MENTAL ARITHMETIC.

PART FIRST.

SECTION I.

MULTIPLICATION OF TENS AND UNITS.

1. A man drove six oxen to market, and sold three of them for 50 dollars apiece. What did they come to?

Three times 50 are 150. *Ans.* 150 dollars.

He sold the remaining three for 52 dollars apiece. What did they come to?

Three times 50 are 150, and three times 2 are 6, which added to 150 makes 156. *Ans.* 156 dollars.

What did they all come to?

Twice 100 is 200, and twice 50 is 100, which added to 200 makes 300, and 6 added to 300 makes 306. *Ans.* 306 dollars.

2. A merchant bought 45 barrels of flour for 6 dollars a barrel. What did it come to?

6 times 40 are 240, and 6 times 5 are 30; 30 added to 240 makes 270. *Ans.* 270 dollars.

He bought 75 barrels more at 5 dollars a barrel. What did it come to?

5 times 70 are 350; 5 times 5 are 25, which added to 350 makes 375. *Ans.* 375 dollars.

What did all the flour come to?

300 and 200 are 500; 70 and 70 are 140, which added to 500 makes 640, and 5 are 645. *Ans.* 645 dollars.

3. What will 87 barrels of flour come to at 6 dollars a barrel?

6 times 80 are 480; and 6 times 7 are 42, which added to 480 makes 522. *Ans.* 522 dollars.

4. What are 7 times 68? What are 8 times 72?
What are 9 times 84? What are 4 times 96?
8 times 64? 5 times 72? 7 times 83? 5 times 79?
4 times 98? 3 times 81? 6 times 73? 6 times 86?

The preceding examples will show the importance of being able readily to multiply tens by units. This becomes easy after acquiring the Multiplication Table. It may be connected with a review of the Multiplication Table in the following manner:

Twice 1 are how many?		Twice 10 are how many?	
Twice 2 are how many?		Twice 20 are how many?	
Twice 3?	Twice 30?	Twice 4?	Twice 40?
Twice 5?	Twice 50?	Twice 6?	Twice 60?
Twice 7?	Twice 70?	Twice 8?	Twice 80?
Twice 9?	Twice 90?	Twice 10?	Twice 100?
3 times 1?	3 times 10?	3 times 2?	3 times 20?
3 times 3?	3 times 30?	3 times 4?	3 times 40?
3 times 5?	3 times 50?	3 times 6?	3 times 60?
3 times 7?	3 times 70?	3 times 8?	3 times 80?
3 times 9?	3 times 90?	3 times 10?	3 times 100?
4 times 1?	4 times 10?	4 times 2?	4 times 20?
4 times 3?	4 times 30?	4 times 4?	4 times 40?
4 times 5?	4 times 50?	4 times 6?	4 times 60?
4 times 7?	4 times 70?	4 times 8?	4 times 80?
4 times 9?	4 times 90?	4 times 10?	4 times 100?
5 times 1?	5 times 10?	5 times 2?	5 times 20?
5 times 3?	5 times 30?	5 times 4?	5 times 40?
5 times 5?	5 times 50?	5 times 6?	5 times 60?
5 times 7?	5 times 70?	5 times 8?	5 times 80?
5 times 9?	5 times 90?	5 times 10?	5 times 100?

6 times 1?	6 times 10?	6 times 2?	6 times 20?
6 times 3?	6 times 30?	6 times 4?	6 times 40?
6 times 5?	6 times 50?	6 times 6?	6 times 60?
6 times 7?	6 times 70?	6 times 8?	6 times 80?
6 times 9?	6 times 90?	6 times 10?	6 times 100?
7 times 1?	7 times 10?	7 times 2?	7 times 20?
7 times 3?	7 times 30?	7 times 4?	7 times 40?
7 times 5?	7 times 50?	7 times 6?	7 times 60?
7 times 7?	7 times 70?	7 times 8?	7 times 80?
7 times 9?	7 times 90?	7 times 10?	7 times 100?
8 times 1?	8 times 10?	8 times 2?	8 times 20?
8 times 3?	8 times 30?	8 times 4?	8 times 40?
8 times 5?	8 times 50?	8 times 6?	8 times 60?
8 times 7?	8 times 70?	8 times 8?	8 times 80?
8 times 9?	8 times 90?	8 times 10?	8 times 100?
9 times 1?	9 times 10?	9 times 2?	9 times 20?
9 times 3?	9 times 30?	9 times 4?	9 times 40?
9 times 5?	9 times 50?	9 times 6?	9 times 60?
9 times 7?	9 times 70?	9 times 8?	9 times 80?
9 times 9?	9 times 90?	9 times 10?	9 times 100?
10 times 1?	10 times 10?	10 times 2?	10 times 20?
10 times 3?	10 times 30?	10 times 4?	10 times 40?
10 times 5?	10 times 50?	10 times 6?	10 times 60?
10 times 7?	10 times 70?	10 times 8?	10 times 80?
10 times 9?	10 times 90?	10 times 10?	10 times 100?
11 times 1?	11 times 10?	11 times 2?	11 times 20?
11 times 3?	11 times 30?	11 times 4?	11 times 40?
11 times 5?	11 times 50?	11 times 6?	11 times 60?
11 times 7?	11 times 70?	11 times 8?	11 times 80?
11 times 9?	11 times 90?	11 times 10?	11 times 100?
11 times 11?	11 times 110?	11 times 12?	11 times 120?
12 times 1?	12 times 10?	12 times 2?	12 times 20?
12 times 3?	12 times 30?	12 times 4?	12 times 40?
12 times 5?	12 times 50?	12 times 6?	12 times 60?
12 times 7?	12 times 70?	12 times 8?	12 times 80?
12 times 9?	12 times 90?	12 times 10?	12 times 100?
12 times 11?	12 times 110?	12 times 12?	12 times 120?

A number which contains another number a certain number of times, is a multiple of that number.

Thus 6 is a multiple of 2; 15 of 3; 28 of 7.*

Name all the multiples of 2, from 2 to 60.

Name the multiples of 20, from 20 to 600.

What are the multiples of 3 up to 75? Of 30 up to 750?

What are the multiples of 4 up to 80? Of 40 up to 800?

What are the multiples of 5 up to 100? Of 50 up to 1000? Of 6 to 72? Of 60 to 720? Of 7 to 84? Of 70 to 840? Of 8 to 96? Of 80 to 960? Of 9 to 108? Of 90 to 1080? Of 10 to 120? Of 100 to 1200?

SECTION II.

MULTIPLICATION OF TENS AND UNITS.—COMPLEMENT.

1. What will 17 tons of hay come to at 8 dollars a ton?

Ans. 8 times 10 are 80, and 8 times 7 are 56; 56 added to 80 makes 136. 136 dollars.

2. What will 37 pounds of sugar come to at 9 cents a pound?

3. A man drove 87 sheep to market, and sold them for 6 dollars apiece. What did they come to?

4. A man travelled on foot eight days; he travelled 29 miles each day. How many miles did he travel in all?

In each of the above examples, the second product, when added to the first, makes a sum exceeding the next even hundred: thus, in the 1st ex., 80+56; in

---

* See Note 1, at the end of Part First

the 2d, 270+63; in the 3d, 480+42; in the 4th, 160+72.

In order to perform such examples with ease, quickness, and without mistake, each step in the process should be made the subject of distinct practice. To illustrate these steps by the first example, 80+56, the first thing to be done is to think of the number which must be added to 80 to make 100, namely, 20; the next is to take this 20 from the 56, and what remains (36) will belong to the next hundred.

The number which, in such cases, must be added to a given number to make up an even hundred, may be called the *complement* of that number. Thus the complement of 80 is 20; of 60, 40; of 90, 10; of 56, 44. What is the complement of 10? 30? 50? 70?

* What is the complement of

10?	20?	30?	40?	50?	60?	70?	80?	90?
11?	21?	31?	41?	51?	61?	71?	81?	91?
12?	22?	32?	42?	52?	62?	72?	82?	92?
13?	23?	33?	43?	53?	63?	73?	83?	93?
14?	24?	34?	44?	54?	64?	74?	84?	94?
15?	25?	35?	45?	55?	65?	75?	85?	95?
16?	26?	36?	46?	56?	66?	76?	86?	96?
17?	27?	37?	47?	57?	67?	77?	87?	97?
18?	28?	38?	48?	58?	68?	78?	88?	98?
19?	29?	39?	49?	59?	69?	79?	89?	99?

How many are 40+76? 80+34? 70+91? 90+17? 25+83? 36+71? 45+82? 56+73? 43+82? 95+36? 37+84? 45+76? 88+37? 94+17? 76+87?

† How many are

12×2, 3, 4, 5, 6, 7, 8, 9, 10?
13×2, 3, 4, 5, 6, 7, 8, 9, 10?
14×2, 3, 4, 5, 6, 7, 8, 9, 10?
15×2, 3, 4, 5, 6, 7, 8, 9, 10?

How many are

16×2, 3, 4, 5, 6, 7, 8, 9, 10?
17×2, 3, 4, 5, 6, 7, 8, 9, 10?
18×2, 3, 4, 5, 6, 7, 8, 9, 10?
19×2, 3, 4, 5, 6, 7, 8, 9, 10?

* See Note 2.

† Note 3.

How many are
20×2, 3, 4, 5, 6, 7, 8, 9, 10?
21×2, 3, 4, 5, 6, 7, 8, 9, 10?
22×2, 3, 4, 5, 6, 7, 8, 9, 10?
23×2, 3, 4, 5, 6, 7, 8, 9, 10?
24×2, 3, 4, 5, 6, 7, 8, 9, 10?
25×2, 3, 4, 5, 6, 7, 8, 9, 10?
26×2, 3, 4, 5, 6, 7, 8, 9, 10?
27×2, 3, 4, 5, 6, 7, 8, 9, 10?
28×2, 3, 4, 5, 6, 7, 8, 9, 10?
29×2, 3, 4, 5, 6, 7, 8, 9, 10?
30×2, 3, 4, 5, 6, 7, 8, 9, 10?
31×2, 3, 4, 5, 6, 7, 8, 9, 10?
32×2, 3, 4, 5, 6, 7, 8, 9, 10?
33×2, 3, 4, 5, 6, 7, 8, 9, 10?
34×2, 3, 4, 5, 6, 7, 8, 9, 10?
35×2, 3, 4, 5, 6, 7, 8, 9, 10?
36×2, 3, 4, 5, 6, 7, 8, 9, 10?
37×2, 3, 4, 5, 6, 7, 8, 9, 10?
38×2, 3, 4, 5, 6, 7, 8, 9, 10?
39×2, 3, 4, 5, 6, 7, 8, 9, 10?
40×2, 3, 4, 5, 6, 7, 8, 9, 10?
41×2, 3, 4, 5, 6, 7, 8, 9, 10?
42×2, 3, 4, 5, 6, 7, 8, 9, 10?
43×2, 3, 4, 5, 6, 7, 8, 9, 10?
44×2, 3, 4, 5, 6, 7, 8, 9, 10?
45×2, 3, 4, 5, 6, 7, 8, 9, 10?
46×2, 3, 4, 5, 6, 7, 8, 9, 10?
47×2, 3, 4, 5, 6, 7, 8, 9, 10?
48×2, 3, 4, 5, 6, 7, 8, 9, 10?
49×2, 3, 4, 5, 6, 7, 8, 9, 10?

How many are
50×2, 3, 4, 5, 6, 7, 8, 9, 10?
51×2, 3, 4, 5, 6, 7, 8, 9, 10?
52×2, 3, 4, 5, 6, 7, 8, 9, 10?
53×2, 3, 4, 5, 6, 7, 8, 9, 10?
54×2, 3, 4, 5, 6, 7, 8, 9, 10?
55×2, 3, 4, 5, 6, 7, 8, 9, 10?
56×2, 3, 4, 5, 6, 7, 8, 9, 10?
57×2, 3, 4, 5, 6, 7, 8, 9, 10?
58×2, 3, 4, 5, 6, 7, 8, 9, 10?
59×2, 3, 4, 5, 6, 7, 8, 9, 10?
60×2, 3, 4, 5, 6, 7, 8, 9, 10?
61×2, 3, 4, 5, 6, 7, 8, 9, 10?
62×2, 3, 4, 5, 6, 7, 8, 9, 10?
63×2, 3, 4, 5, 6, 7, 8, 9, 10?
64×2, 3, 4, 5, 6, 7, 8, 9, 10?
65×2, 3, 4, 5, 6, 7, 8, 9, 10?
66×2, 3, 4, 5, 6, 7, 8, 9, 10?
67×2, 3, 4, 5, 6, 7, 8, 9, 10?
68×2, 3, 4, 5, 6, 7, 8, 9, 10?
69×2, 3, 4, 5, 6, 7, 8, 9, 10?
70×2, 3, 4, 5, 6, 7, 8, 9, 10?
71×2, 3, 4, 5, 6, 7, 8, 9, 10?
72×2, 3, 4, 5, 6, 7, 8, 9, 10?
73×2, 3, 4, 5, 6, 7, 8, 9, 10?
74×2, 3, 4, 5, 6, 7, 8, 9, 10?
75×2, 3, 4, 5, 6, 7, 8, 9, 10?
76×2, 3, 4, 5, 6, 7, 8, 9, 10?
77×2, 3, 4, 5, 6, 7, 8, 9, 10?
78×2, 3, 4, 5, 6, 7, 8, 9, 10?
79×2, 3, 4, 5, 6, 7, 8, 9, 10?

To multiply any number less than 10 by 11, repeat the figure expressing the number; as, 3 times 11 is 33, $4 \times 11 = 44$.

To multiply by 11 any number of two figures. Think of the first figure, then of the sum of the two

figures, then of the last figure. These three figures will express the answer. Thus, 11×23; the first, 2; the sum of the two, 5; the last, 3. *Ans.* 253. $11\times24=264$, $11\times32=352$, $11\times43=473$.

Remember, if the sum of the two is as much as 10, you must increase the first figure by 1.

How many are 11×26? 11×28? 11×29? 11×41? 11×43? 11×45? 11×61? 11×62? 11×64? 11×71? 11×73? 11×81? 11×94? 11×75? 11×86? 11×89? 11×82? 11×84?

SECTION III.

PRACTICAL QUESTIONS.

1. If a railroad car travels 23 miles in one hour, how far will it travel in 9 hours?

2. If a horse travels 38 miles in one day, how far will he travel in 6 days?

3. If a man earns 14 dollars a month, how much will he earn in 7 months?

4. If a man spends 6 cents a day for ardent spirit, how much will that amount to in 10 days? How much in 30 days? How much in 300 days? How much in 60 days? How much in 5 days? How much in 365 days?

5. If a man earns 10 cents in an hour, and works 12 hours in a day, how much will he earn in a week, there being 6 working days in a week? How much in 10 weeks? How much in 50 weeks?

6. If a scholar in school is idle 18 minutes in the forenoon, and 18 minutes in the afternoon, how much time will he lose in a week, if there are 6 forenoons and 4 afternoons of school time in a week?

7. If a town is 6 miles long, and 5 miles broad, how

many square miles does it contain? If there are 40 inhabitants on every square mile, how many inhabitants does the town contain? 40 times 30. 4 times 30 are 120. 40 times 30 are 10 times as many. If one in 12 of the inhabitants were able-bodied men, how many able-bodied men would there be? If one in 6 are able-bodied men, how many such are there?

8. What will 146 yards of broadcloth come to at 5 dollars a yard?

9. What will 86 yards of broadcloth come to at 6 dollars and a half a yard?

10. What will 740 barrels of flour come to at 6 dollars a barrel? At 5 dollars a barrel? At 5 dollars and a half a barrel?

11. What will 33 gallons of molasses come to at 31 cents a gallon? At 34 cents a gallon? At 40 cents a gallon?

12. What will 38 pounds of coffee come to at 14 cents a pound? At 16 cents a pound?

13. If there are 12 windows in the front of a house, 10 in the rear, and 6 in each end, each window containing 12 lights; how many lights are there in the front of the house? How many in the rear? How many in both ends? How many in all? If each light cost 10 cents, how much would they all cost? If the setting of each light cost 5 cents, how much would the setting of them all cost?

14. There are 8 houses in a row: each house has 8 windows in front, 7 in the rear, and 5 in each end, each window containing 12 lights; how many lights are there in each house? How many in all the houses? What would be the cost of them all at 7 cents for each light? What would be the cost of setting them all at 3 cents for each light? At the above price; what would be the cost of the glass and the setting of it for one of the houses? What would be the cost of the glass and the setting of it for two of the houses? For three of the houses? For four of the houses? For five of the houses?

15. What are $8\frac{1}{2}$ tons of hay worth, at 13 dollars a ton?

16. If one acre of ground produce 65 bushels of corn, how much would grow on 9 acres?

17. If an acre of ground produce 228 bushels of potatoes, how many bushels would grow on 5 acres?

18. If standing wood is worth 2 dollars a cord, what is the value of the wood on 7 acres, each of which furnishes 18 cords?

19. If there are 200 families in a town, and each family consumes 12 cords of wood annually, how many cords are used in the town each year?

What is the whole value of the wood, at $3\frac{1}{2}$ dollars a cord? How much money will be saved in the town if each family burns 2 cords less than before?

SECTION IV.

DIVISION.

1. What is one half of 20? Of 40? Of 60? Of 80? Of 100? Of 120? Of 140? Of 160?

2. What is one half of 22? Of 42? Of 62? Of 82? Of 102? Of 112? Of 122? Of 142? Of 162? Of 182?

3. What is one half of 44? 64? 86? 48? 66? 28? 84? 68? 46? 24? 26? 62?

4. What is one half of 70? Divide it into 60 and 10.

What is one half of 90? Divide it into 80 and 10.

What is one half of 50? Of 30? Of 110? Of 130? Of 150?

5. What is one half of 32? Of 54? Divide it into 50 and 4.

What is one half of 76? Of 74? Of 78? Of 96? Of 98? Of 92? Of 94? Of 72? Of 76? Of 58? Of 56?

6. What is one half of 43? One half of 40 is 20. One half of 3 is 1½; this added to 20 makes 21½.

What is one half of 47? Of 49? Of 63? Of 65? Of 67? Of 69? Of 83? Of 85? Of 87? Of 89?

7. What is one half of 33? Divide into 30 and 3.

What is one half of 35? 37? 39? 51? 53? 55? 57? 59? Of 71? Of 73? Of 75? Of 77? Of 79? Of 91? Of 93? Of 95? Of 97? Of 99?

8. What is one half of 367? Divide into 300, 60, and 7.

What is one half of 674? Of 895? Of 724? Of 632? Of 945? Of 424? Of 688? Of 546? Of 392?

We can now find a very quick way of multiplying any number by 5. Take one half the number: multiply that by 10. We will take the numbers in question 2, and multiply them by 5 in this way.

Multiply 22 by 5. Half of 22 is 11, and ten times 11 is 110.

Multiply 42 by 5. Half of 42 is 21; ten times that is 210.

Multiply 62 by 5. Half of 62 is 31; 310.
Multiply 82 by 5. Half is 41; 410.
Multiply 102 by 5. Half is 51; 510.
Multiply 112 by 5. Half is 56; 560.
Multiply 122 by 5. Half is 61; 610.
Multiply 142 by 5. Half is 71; 710.
Multiply 162 by 5. Half is 81; 810.
Multiply 182 by 5. Half is 91; 910.

9. Multiply by 5, in this way, the numbers in question 3; 44; 64; 86; 48; 66; 28; 84; 68; 46; 24; 26; 62.

You can, if you wish, perform these examples by both methods, and thus prove the work correct.

Multiply 862 by 5. Half is 431; 4310.—Multiply 672 by 5. Half is 336; 3360.

10. Multiply 686 by 5; 748 by 5; 932 by 5; 896 by 5; 1262 by 5.

If the number to be multiplied is an odd number, so that half of it will show the fraction ½, this, when you multiply by 10, will become 5; for ten halves are 5.

Multiply 781 by 5. Half is 390½; 3905, *Ans.*

11. Multiply 963 by 5. Half is 481½; 4815.— Multiply 845 by 5; 381 by 5; 953 by 5; 845 by 5; 637 by 5; 429 by 5.

12. What is one fourth of 40? One fourth of 80? One fourth of 120? One fourth of 12 is 3; a fourth of 120 is 10 times as much; 30.—What is one fourth of 160? One fourth of 200? Of 240? Of 280? Of 320? Of 360? Of 400.

13. What is one fourth of 60? Take half of it; then half of that half. Half of 60 is 30; half of 30 is 15.—What is one fourth of 100? One fourth of 140? One fourth of 180? One fourth of 220? One fourth of 260? One fourth of 300?

14. Another way of finding one fourth of the numbers in the last example, is as follows:

What is one fourth of 60? Divide 60 into 40 and 20. One fourth of 40 is 10; one fourth of 20 is 5; 15.

What is one fourth of 100? Divide into 80 + 20.

What is one fourth of 140? Divide into 120 + 20.

What is one fourth of 180? Divide into 160 + 20, &c.

15. What is one fourth of 30? Of 50? Of 70?

Find the best way of answering these, for yourself.

What is one fourth of 90? Of 110? Of 130? Of 150? Of 170? Of 190? Of 210? Of 230? Of 250.

16. What is one fourth of 76? Divide the number into 40 and 36.—What is one fourth of 96? Divide the number into 80 and 16.

What is one fourth of 52? Of 64? Of 84?

17. What is one fourth of 368? There are several ways of dividing this number; first, into 200 + 100 + 60 + 8; a second way would be, into 200 + 160 + 8;

another way, into 320 + 48. This is shorter than either of the former. A better division still is into 360 + 8.

What is one fourth of 496? Into what different sets of numbers, each divisible by 4, can you divide this? What is one fourth of 964? Of 336? Of 836? 596? 472? 1324? 1728? 2236?

18. What is one tenth of 10? Of 20? Of 30? Of 40? Of 50? Of 60? 70? 80? 90? 100? 110? 120? 130? 140? 150?

19. What is one tenth of 5? *Ans.* 5 tenths of 1, or $\frac{5}{10}$, equal to $\frac{1}{2}$.

What, then, is one tenth of 15? Of 25? 35? 45? 55? 65? 75? 85? 95? 14? 17? 36? 47? 52? 91? 43? 28? 65? 86? 47?

20. What is one fifth of 25? 40? 45? 50? 55? 60? 65? 70? Divide 70 into 50 and 20.—Of 75? Divide into 50 and 25.—Of 80? Divide into 50 and 30.—Of 85? Of 90? Of 95? Of 100?

21. What is one fifth of 64? Of 82? Of 91? Of 67? Of 73? Of 59? Of 63? Of 72? Of 78? Of 83? Of 87?

22. What is one fifth of 140? Of 385? Of 260? Of 480? Of 390? Of 580? Of 470? Of 865? Of 395?

23. The following is a short way of dividing a number by 5. Take one tenth of the number, and double it. That, of course, gives 2 tenths, which is equal to 1 fifth. Take the numbers in the last example, and divide by 5 in this way. One fifth of 140; one tenth is 14; double that is 28. One fifth of 385; one tenth is 38 and 5 tenths; twice that is 77. One fifth of 260; one tenth is 26; twice 26 is 52. One fifth of 480; one tenth is 48; twice that is 96. What is one fifth of 390? Of 580? 470? 865? 395?

24. The following is a short way of multiplying a number by 25. Take one fourth of the number;

multiply that by 100. This will give 100 fourths, which are equal to 25 whole ones.

Multiply 40 by 25. One fourth is 10; one hundred times that are 1000. — Multiply 60 by 25. One fourth is 15; 1500.

Multiply 80 by 25. One fourth is 20; 2000.
Multiply 120 by 25. A fourth is 30; 3000.
Multiply 112 by 25. A fourth is 28; 2800.
Multiply 116 by 25. A fourth is 29; 2900.

25. Multiply 22 by 25. One fourth is 5 and a half; 100 times this are 5 hundred and half a hundred; 550.

Multiply 26 by 25. One fourth is $6\frac{1}{2}$; 650.
Multiply 28 by 25. One fourth is 7; 700.

Multiply 30 by 25; 32 by 25; 34 by 25; 36 by 25; 40 by 25.

Multiply 42 by 25; 44 by 25; 46 by 25; 48 by 25; 50 by 25.

26. Multiply 13 by 25. One fourth is 3 and one fourth; one hundred times this is 300 and one fourth of a hundred or 25; 325.

Multiply 15 by 25. One fourth is 3 and three fourths; one hundred times this are 3 hundred and 3 fourths of a hundred or 75; 375.

Multiply 17 by 25; 19 by 25; 21 by 25; 23 by 25; 27 by 25; 29 by 25; 31 by 25; 33 by 25; 35 by 25.

27. Multiply 116 by 25. One fourth is 29; 2900.
Multiply 117 by 25. One fourth is $29\frac{1}{4}$; 2925.
Multiply 121 by 25; 87 by 25; 156 by 25; 960 by 25.

28.* What is one third of 60? Of 90? Of 120? Of 15? Of 150? Of 45? Of 450?

What is one third of 18? Of 180? Of 21? 210? Of 36? Of 360? Of 30? Of 390?

What is one third of 72? Divide into 60 and 12.

What is one third of 54? Of 85? Of 98?

What is one sixth of 60? Of 80? Divide into 60 and 20. — Of 74? Of 84? Of 96? Of 100?

---

* Note 4.

What is one sixth of 12? Of 120? Of 130? Of 140? Of 144?

What is one sixth of 18? Of 180? Of 200? Of 210? Of 220?

What is one sixth of 384? Of 492? Divide into 480 and 12.—Of 555? Divide into 540 and 15.—Of 620? Of 726? Of 947?

29. What are the two factors of 18? Of 180?

What are the two factors of 27? Of 270? Of 22? Of 220? Of 35? Of 350? Of 54? Of 540? Of 45? Of 450? Of 21? Of 210? Of 28? Of 280? Of 42? Of 420?

30. What two numbers, multiplied together, will produce 24?

What other two factors will produce 24? What other two?

What two factors will produce 240? What other two? What others?

What two factors will produce 30? What others?

What two factors will produce 300? What others?

What two factors will produce 18? What others?

What two will produce 180? What others?

Name all the pairs of factors that will produce 36; 360; 48; 480; 60; 600; 64; 640; 72; 720.

31. What is one ninth of 27? A man divided 270 dollars equally among 9 persons. How much did he give to each?

32. What is one fourth of 48? If 480 dollars are divided into 4 equal shares, what will each share be?

What is one eighth of 480? One sixth of 480? One twelfth of 480?

33. What is one seventh of 63? If a ship sails, at a uniform rate, 630 miles in a week, how many miles does she sail in a day?

What is one ninth of 630? What is one sixth of 630? What is one third of 630?

34. What is one fifth of 25? If 250 trees are

placed in 5 equal rows, how many will there be in each row?

If placed in 50 equal rows, how many would there be in each row?

35. What is one fourth of 36? In a circle there are 360 degrees. How many are there in one fourth of a circle? How many in one eighth of a circle? How many in one sixteenth of a circle?

36. What is one eighth of 56? If 560 trees were planted in 8 equal rows, how many would there be in each row? If planted in 16 rows, how many would there be in each row?

37. What is one eleventh of 55? If you place 550 trees in 11 equal rows, how many will there be in a row? If you place them in 50 rows, how many will there be in each row? If you place them in 25 rows, how many will there be in each row?

38. What is one twelfth of 96? If a man spends 960 dollars in a year, how much will be his average expense for each month?

39. What is one tenth of 40? Of 400? Of 4000?

What is one fourth of 40? 400? Of 4000? Of 80? Of 800? Of 8000? One fourth of 12? Of 120? Of 1200? Of 12,000?

40. What is one fifteenth of 60? Of 600? Of 6000?

What is one thirtieth of 60? Of 600? Of 6000? Of 1200? Of 12,000?

41. What is one fifth of 92? What is one third of 51? One fourth of 65? One fifth of 78? One sixth of 96? One seventh of 100? Divide into 70 and 30. —What is one ninth of 117? *Ans.* One ninth of 90 is 10; one ninth of 27 is 3; 10 and 3 are 13.

42. What is one third of 49? One sixth of 84? One fifth of 79? One eighth of 100? One seventh of 91? One sixth of 79? One fourth of 76? Of 92? Of 57? Of 60? Of 52? Of 65? Of 70?

43. What is one fourth of 480? What is one fifth of 155? Divide into 150 and 5.—What is one fourth of 920? What is one fifth of 15,765? This number may be divided into 15,000, 750, and 15; or 15,000, 500, 250, and 15.

44.* What is one sixth of 4836? One eighth of 336? Divide into 320 and 16.—What is one seventh of 574? One third of 684? One sixth of 43,248? One ninth of 72,108? Of 64,827? One fifth of 5275? One fourth of 92,648?

45. What is one third of 6156? Of 8436?

46. What is one fourth of 6428? Of 9648?

47. What is one fifth of 7655? Of 12,535?

48. What is one sixth of 13,218? Of 1944?

49. What is one seventh of 10,542? Of 14,280?

50. What is one eighth of 1632? Of 2560?

SECTION V.

TABLE OF TIME.

60 seconds, (sec.)	make	1 minute,	marked	m.
60 minutes,		1 hour,		h.
24 hours,		1 day,		d.
7 days,		1 week,		w.
4 weeks,		1 month,		mo.
52 weeks, 1 day, 6 hours,		1 year,		y.
365 days, 6 hours,		1 year,		y.
12 calendar months,		1 year,		y.

In common reckoning, 4 weeks are called a month; but this is merely for convenience in doing business.

* Note 5.

The number of days in a calendar month is 30 or 31; except February, which has 28 days, and in leap year 29. The 6 hours over and above the 365 days in a year, will in 4 years amount to a whole day; it is then added to February, making 29 days, and that year is called *leap year*. The number of days in the other months may be seen in the line below. The months connected by a tie drawn over the words have 31 days; those connected by a tie underneath have 30.

Jan. Feb. March. April. May. June. July. August. Sept. Oct. Nov. Dec.
28. 29.

You observe that, beginning with January, every alternate month has 31 days, till you come to July and August. Here there are two months together that have 31, and then the alternation goes on as before to the end of the year.

The leap year may be easily known from the fact that the number of the year is exactly divisible by 4. Thus 1844 was leap year; the number can be divided by 4.

What years in the present century have been leap years? What years will be leap years from now to the close of the century?

1. In one minute there are 60 seconds; in one hour there are 60 minutes. How many seconds are there in one hour? How many in 10 hours? How many in 20 hours? How many in one day? How many in seven days, or one week? How many in ten days? In 100 days? In 300 days? In 350 days? In 365 days?

2. If you save 30 minutes from idleness each day, how many hours will you save in a week? How many in 5 weeks? How many in 50 weeks? How many in 52 weeks?

3. If you read 40 pages each day, how many

pages will you read in one week? How many in 10 weeks? How many in 52 weeks?

4. If a printer sets 4 pages of type in a day, in how many days will he set the type for a book of 500 pages? What will his wages come to, at $1.50 a day?

5. If there are 300 members in the Legislature of Massachusetts, and each member receives 2 dollars a day during the session, what does the pay of all the members come to for one day? What does the pay of the Legislature amount to for one week? For 10 weeks?

6. The number of members in Congress is about 275. At 8 dollars a day, what is the amount of their pay each day? What would be the amount of their pay for 10 days? For 100 days?

7. How many days are there in the 3 months of spring? How many days in the 3 months of summer? How many days in autumn?

8. How many days in the winter of leap year? How many days were there in the winter of 1844? How many days in the winter of 1845?

9. If January comes in on Monday, on what day of the week will February come in?

If March comes in on Wednesday, on what day of the week will April come in?

If August comes in on Saturday, on what day of the week will September come in?

10. If April comes in on Sunday, on what day of the week will it go out?

If June comes in on Tuesday, on what day will it go out?

If September comes in on Saturday, on what day will it go out?

11. If January comes in on Friday, how many Sundays will there be in that month?

If it comes in on Thursday, how many Sundays will there be in that month?

If June comes in on Friday, how many Sundays will there be in that month? If it comes in on Saturday, how many?

If February comes in on Saturday, and that year is leap year, how many Saturdays will there be in the month? If it is not leap year, how many?

In 1845, February came in on Saturday. How many Saturdays were there in that month?

In 1844, February came in on Thursday. How many Thursdays were there in that month?

12. If January comes in on Monday, on what day of the week will March come in, if it is leap year? On what day, if it is not leap year?

13. If June comes in on Wednesday, what day of the week will the 1st of August be? The 9th? The 12th? The 15th?

TABLE OF LINEAR MEASURE.

12	inches, (in.) . make	. 1	foot, . . . marked	. . . ft.
3	feet, ...	. 1	yard, ...	yd.
$5\frac{1}{2}$	yards, $16\frac{1}{2}$ feet, ...	. 1	rod, ...	rd.
40	rods, ...	. 1	furlong, ...	fur.
8	furlongs = 320 rods,	. 1	mile, ...	m.
3	miles, ...	. 1	league, ...	l.
$69\frac{1}{2}$	miles, ...	. 1	degree of latitude, . .	deg.

For lengths less than an inch, the inch is divided into fourths, eighths, tenths, or twelfths.

1. How many inches in 2 feet? In 4 feet? In 5 feet? In 7 feet? In 10 feet? In 12 feet? How many inches in 4 yards? In 1 rod? In 3 rods? How many feet in 1 furlong? In 2 furlongs? In 4 furlongs? In 1 mile?

2. How many miles in 46 leagues? In 132 leagues? How many miles in 2 degrees of latitude? In 3 degrees? In 4½ degrees? In 6 degrees?

In estimating the miles in any number of degrees of latitude, it is most convenient to call a degree 70 miles; and then, if we wish to be accurate, we may subtract from the answer half as many miles as there are degrees. In this way, the distance of places from each other may be determined on a map. The degrees of latitude on the margin may be used as a scale of miles. If the distance of two places from each other is equal to 6½ degrees of latitude, how many miles are they apart?

3. How many yards in 10 rods? In 20 rods? In 30 rods? In 1 furlong? In 8 furlongs, or 1 mile?

4. In measuring land or a road with a chain 4 rods long, how many times must the chain be applied to the ground in measuring one mile? How many times in measuring the road from Boston to Salem, 15 miles? How many in measuring from Boston to Providence, 40 miles?

5. If a man walks 3 miles in an hour, how many minutes will he be in walking 1 mile? How many minutes in walking 1 fourth of a mile? How many rods will he walk in 1 minute? *Ans.* 16.

How many seconds will he be, then, in walking 1 rod? 16 will go into 60, 3 times and 12 over. He will be, then, a little less than 4 seconds in walking 1 rod.

Let us now suppose he is precisely 4 seconds in walking 1 rod; how many rods would he walk in a minute? How many in 10 minutes? How many in 60 minutes? How many miles?

6. If a man in walking takes 6 steps to a rod, how many steps will he take in walking a mile? How many in walking 10 miles? How many in walking 40 miles?

7. If a man in walking takes 6 steps to a rod, and takes 2 steps in a second, how many seconds will he be in walking one rod? How many seconds in walking 10 rods? 20 rods? If a man walks 20 rods in one minute, how many minutes will it take him to walk a mile? 20 are contained in 320 just as many times as 2 are contained in 32.

8. If a man walks 20 rods in one minute, how long will it take him to walk 4 miles?

9. If a railroad train goes 30 miles in an hour, how far does it go in one minute? How many rods in one second?

Ans. 30 miles in 60 minutes is 1 mile in 2 minutes; half a mile in one minute; quarter of a mile in half a minute; that is 80 rods in 30 seconds; that is 8 rods in 3 seconds; and in 1 second, one third of 8 rods, or 2 rods and two thirds.

10. How many rods in 14 miles?

In 1 rod there are 16½ feet. In 1 mile there are 320 rods. How many feet are there in a mile? There are various ways of finding the answer to this question; some of them will be suggested, and the pupil left to take his choice.

First, how many feet are there in 300 rods?

This is not difficult; for in 3 rods there are three times 16½ feet, and in 300 rods there are 100 times as many. 3 times 15 feet are 45 feet; 3 times 1½ feet are 4½, which added to 45 make 49½ feet in 3 rods. Now, 100 times 49 are 4900, and 100 halves are 50; 4950 feet in 300 rods. In 20 rods there are ten times as many feet as in 2 rods; in 2 rods there are twice 16½ or 33; in 20 rods, therefore, there are 330 feet; 300 added to 4900 make 5200, and 30 added to 50 make 80; there are, then, 5280 feet in a mile.

Another method would be to multiply 320 first by 8, and that product by 2, for 8 and 2 are the factors of 16; then, as there was ½ a foot in each rod left out,

there must be added half as many feet as there are rods, or half of 320.

Another method would be to multiply 320 by 10, then by 6, and add the products, and lastly by $\frac{1}{2}$, and add that to the other products.

The pupil can try each of these ways, and see if he obtains the same answer.

Let us now see if our answer is correct. If there are 5280 feet in a mile, how many are there in half a mile? One half of 5200 is 2600; one half of 80 is 40; there are, then, 2640 feet in half a mile. How many in 1 fourth of a mile? One half of 2640 feet, which is 1320. Now, 1 fourth of a mile is 80 rods. If, then, there are 1320 feet in 80 rods, how many will there be in 8 rods? One tenth as many. One tenth of 1320 is 132. Now, how many are there in one rod? One eighth of 132; dividing 132 into 80 and 52; one eighth of 80 is 10, and one eighth of 52 is $6\frac{4}{8}$ or $\frac{1}{2}$, which added to 10 make $16\frac{1}{2}$. We have now come down from 5280, and arrived by successive divisions to $16\frac{1}{2}$, the number from which we started at first. The answer is thus proved to be correct.

11. How many feet are there in 2 rods? In 20 rods? In 200 rods?

12. How many feet are there in 3 rods? In 30 rods? In 300 rods? In 8 rods? In 80 rods, or a quarter of a mile?

13. How many rods are there in 2 miles? In 4 miles? In 8 miles? In 20 miles? In 30 miles? In 50 miles?

14. How many rods are there in half a mile? In three fourths of a mile? In 1 mile and a half? In 1 mile and 3 furlongs? In 2 miles and 5 furlongs? In 4 miles and 7 furlongs?

15. How many yards are there in 2 rods? In 20 rods? In 3 rods? In 30 rods? In 300 rods?

How many yards are there in 3 rods and 4 feet? How many yards are there in 17 rods and 11 feet?

16. How many inches are there in 7 feet? In 9 feet? In 6 feet? In 8 feet and 6 inches? In 11 feet 9 inches?

17. How many inches are there in 1 rod, or 16½ feet? How many inches in 2 rods? In 3 rods? In 4 rods?

18. A house is 46 feet and 5 inches in length. How many inches long is it?

A creeping vine grows on an average 3 inches a day. How many days will it take to grow from the ground to the top of a house that is 25 feet high?

19. A stage-horse travels 13 miles and 20 rods each day. How far will he travel in 60 days? How far in 120?

20. If a horse travels 16 miles in a day, how many miles will he travel in 5 days? In 10 days? In 15 days? In 20 days? In 200 days? In 300 days?

21. What is the weight of iron used in one mile of railroad, allowing 50 pounds for a yard of rail?

If one yard of rail weighs 50 pounds, the two rails for one yard of the road would weigh 100 pounds: from this may be obtained the weight for one rod; for 10 rods; for 100 rods; for 300 rods; for 320 rods.

22. If a yard of iron rail weighs 75 pounds, what would be the weight of the two rails of a single track for one rod? For 2 rods? For 3 rods? For 300 rods? For 20 rods? For 200 rods?

What would be the cost of the rail for one rod, at 2 cents a pound? At 3 cents a pound? At 4 cents a pound?

23. If the cost of the iron for a single track of railroad is 6000 dollars a mile, and the cost of the land and the labor of construction equals that of the iron, what would be the cost of 15 miles of railroad? Of 24 miles?

24. What would be the cost of constructing 1 mile of common road at 225 dollars a rod?

25. What would be the cost of building 80 rods of common wall at 54 cents a rod?

26. If a horse travels 10 miles in an hour, how long is he in travelling 1 mile? How long in travelling $\frac{1}{4}$ of a mile, or 80 rods? How many seconds is he in travelling 8 rods? How long in travelling 1 rod?

27. A body falling through the air, falls in the first second $16\frac{1}{2}$ feet, and in each succeeding second it falls twice $16\frac{1}{2}$ feet farther than in the preceding second. How far would a stone fall in 2 seconds?

28. How far would it fall in the third second? How far would it fall in 3 seconds?

29. How far would it fall in the fourth second? How far would it fall in 4 seconds?

30. Sound moves through the air at the rate of 1090* feet in a second. How many feet will it move in 3 seconds? How many feet in 4 seconds? How many feet in 5 seconds?

As sound is found thus to pass 5450 feet in 5 seconds, and as there are 5280 feet in a mile, we see that in 5 seconds sound moves 170 feet more than a mile. Now, as 165 feet is just 10 rods, we say, without much error, that sound moves 1 *mile and* 10 *rods* in 5 *seconds*. This is accurate enough for all common purposes, and you will do well to fix it in your memory, and make your calculations from it.

31. How many rods will sound move in 1 second? One fifth of $320 + 10$ rods $= 66$ rods.

32. How many rods in 2 seconds? How many rods in 4 seconds?

Thus, if you watch the stroke of an axe used by some one at a distance, and observe that the sound comes to you one second later than you see the stroke, you may know that the distance is 66 rods. If the sound of a bell comes to you two seconds after the

* Professor Pierce on Sounds.

stroke is given, you must be distant from the bell 132 rods. In these cases, no allowance is made for the transmission of light. You are supposed to see the motion as soon as it occurs. This is not strictly the fact; but the time is so exceedingly small, that it need not be taken into the account.

33. In a still night, a church bell is sometimes heard at the distance of 12 miles. How many seconds, or nearly how many, after the stroke, would the sound be heard at that distance?

34. If the report accompanying a flash of lightning is heard 4 seconds after the flash is seen, how far from the hearer was the discharge? How far, if the time between the flash and the report is 6 seconds? How far, if the time is 8 seconds? How far, if the time is 10 seconds? How far, if the time is 15 seconds?

35. The report of a cannon has, in some instances, been heard at the distance of 100 miles. Allowing that the sound moves one mile in 5 seconds, in how many seconds after the discharge would the report be heard at the distance of 100 miles?

36. By means of a magnetic telegraph, it is possible to communicate intelligence instantly from New Orleans to Boston, a distance of 1500 miles. If this intelligence could be communicated by sound passing through the air, how long would it be in travelling that distance, allowing 5 seconds to a mile?

A ball discharged from a gun moves at first with a greater speed than sound, but it moves slower and slower, and before it is spent the report overtakes it, and passes by it; for sound moves always at the same rate.

37. If a cannon ball moves a mile in 8 seconds, how long would it be in moving 3 miles? How long in moving one fourth of a mile? How long in moving one eighth of a mile? How long in moving $1\frac{3}{4}$ miles?

SECTION VI.

TABLE OF FEDERAL MONEY

10 mills ... make ...	1 cent, ...	marked ...	ct.
10 cents, ...	1 dime, ...		d.
10 dimes, ...	1 dollar, ...		D.
10 dollars, ...	1 eagle, ...		E.

This is established by law as the currency of the United States.

The general mark for Federal Money is $; as, $5.14, five dollars fourteen cents. A period must always be placed between dollars and cents.

1. How many mills in 2 cents? In 10 cents? In 12 cents? In 5½ cents? In 12½ cents? In 36 cents? In 1 dollar?

2. How many cents in 5 dimes? In 11 dimes? In 16 dimes? In 4½ dollars? In 17¼ dollars? In 12¾ dollars?

3. How many dimes in 7 dollars? In 13½ dollars? In 3 eagles? In 56 dollars? In 100 dollars?

4. How many cents in 35 mills? In 180 mills? In 600 mills? How many dimes in 80 cents? In 210 cents? In 740 cents?

5. How many dollars in 350 cents? In 325 cents? In 700 cents? In 850 cents? In 1400 cents? In 1675 cents? In 925 cents?

TABLE OF STERLING MONEY.

4 farthings, (qr.) make	1 penny, ..	marked ..	d.
12 pence, ...	1 shilling, ...		s.
20 shillings, ...	1 pound, ...		£.

This is the currency of Great Britain.

1. How many farthings are there in 3 pence? In 7 pence? In 8 pence? In 10 pence? In 11 pence?

2. How many pence in 2 shillings? In 12 shillings? In 15 shillings? In 18 shillings? In 16 shillings?

3. How many shillings in 4 pounds? In 7 pounds? In 18 pounds? In 36 pounds? In 84 pounds?

4. How many farthings in 1 shilling and 6 pence? In 2 shillings and 6 pence? In 15 shillings and 4 pence?

How many pence in 10 shillings? In 20 shillings? In 2 pounds? In 4 pounds? In 12 pounds?

5. How many farthings in 1 pound? In 5 pounds? In 8 pounds? In 1 pound 2 shillings?

6. How many pence in 45 farthings? In 128 farthings? In 464 farthings? In 1296 farthings? In 648 farthings?

7. How many shillings in 80 pence? In 67 pence? In 372 pence? In 649 pence? In 840 pence?

8. How many pounds in 267 shillings? In 845 shillings? In 432 shillings? In 640 shillings? In 4000 shillings?

9. How many pounds in 890 pence? In 16,000 farthings? In 720 pence? In 1200 pence? In 456 pence?

10. How many pence in 5 pounds 4 shillings? In 7 pounds 8 shillings? In 12 pounds 3 shillings?

How many farthings in 4 shillings 6 pence? In 9 shillings? How many farthings in 6 pounds 3 shillings 8 pence?

11. A man set out on a journey with £4 8s. 6d. in his pocket. Before spending any thing, he received in payment of a debt £2 3s. 8d. How much had he then? When he arrived home, he had spent £1 4s. 6d. How much had he then?

These denominations, you must bear in mind, have not the same value in English currency that they have in the United States.

In our country, they have different values in the different States, but in none of them so high a value as in England. In the New England States, a shilling is equal to 16 cents and two thirds, and 6 shillings make a dollar. In New York, 12 and a half cents are a shilling, and 8 shillings a dollar. In other States, the values are still different. But these denominations are gradually giving way to those of the Federal currency. They are now used only in naming prices. Accounts are not kept in them, and all that is important in them may be learned by practice without further notice here.

In the Sterling currency, used in England, a pound is equal to 4 dollars, 44 cents, and 4 mills; 10 shillings, therefore, or half a pound, are 2 dollars, 22 cents, 2 mills; and 1 shilling is one tenth part of that, or 22 cents 2 mills. An English sixpence is, therefore, 11 cents 1 mill. The following table will be useful in exchanging English money to our own.

1	pound (£) is	$4.44 4
10	shillings, or half a pound,	2.22 2
1	shilling,	22 2
6	pence, or half a shilling,	11 1
4	shillings 6 pence,	1.00 0
1	guinea, 21 shillings,	4.66 6

The actual value of the English money is a little higher than is here stated, but this is sufficiently accurate for a general table.

1. What is the value, in dollars and cents, of 2£? 3£? 4£? 5£? 1£ 6 s.? 2£ 8 s.? 3 s. 6 d.? 5 s. 9 d.?

TABLE OF DRY MEASURE.

2 pints (pt.)	make	1 quart,	marked qt.
4 quarts,		1 gallon,	gal.
8 quarts,		1 peck,	pk.
4 pecks,		1 bushel,	bu.
8 bushels,		1 quarter,	qr.
36 bushels,		1 chaldron,	ch.

These denominations are used for measuring grain, fruit, and coal. The pint, quart, and gallon, are larger than the same denominations in wine measure, and less than those of beer measure.

1. How many pints in 1 peck? In 3 pecks? In 1 bushel? In 3 bushels? In 4 bushels?

2. How many quarts are there in 1 bushel? In 4 bushels? How many pecks in 7 quarters? In 2 chaldrons?

3. If a horse eat 4 quarts of oats each day, how many bushels will he eat in 10 weeks? How many bushels in 50 weeks? In 52 weeks?

What will they cost at 50 cents a bushel?

4. In 80 quarts how many pecks? How many bushels?

In 644 quarts how many pecks? How many bushels?

In 7840 quarts how many pecks? How many bushels?

5. In 100 pints how many pecks? How many bushels?

In 620 pints how many pecks? How many bushels?

TABLE OF AVOIRDUPOIS WEIGHT.

16 drams (dr.)	make	1 ounce,	marked . . oz.
16 ounces,		1 pound,	lb.
25 pounds,		1 quarter, (net wt.)	qr.
28 pounds,		1 quarter, (gross wt.)	qr.
4 quarters,		1 hundred weight,	cwt.
20 hundred weight,		1 ton,	T.

These denominations are used in weighing hay, grain, meat, flour, and all the most common articles bought and sold by weight. On account of the waste in handling such articles, their shrinking in drying, and worthless admixtures sometimes found in them, 112 pounds are sometimes allowed for one hundred weight; this makes 28 pounds one quarter, and is called *gross weight*. In all the following questions of avoirdupois weight, understand gross weight, unless net weight is expressed.

1. How many drams in 3 oz.? In 5 oz.? In 8 oz.? In 11 oz.? How many oz. in 12 lbs.? In 15 lbs.? In 20 lbs.? In 32 lbs.? In 45 lbs.?

2. How many lbs. in 4 cwt. net weight? In 4 cwt. gross? In 6 cwt. net weight? In 6 cwt. gross? In 5 cwt. 2 qrs. net? In 5 cwt. 2 qrs. gross? In 7 cwt. 3 qrs. net weight? In 7 cwt. 3 qrs. gross?

3. How many lbs. in a ton net weight? In a ton gross? How many lbs. in 5 tons 3 cwt. net? In 5 tons 3 cwt. gross?

4. There are 2 loads of hay whose net weight is as follows: the first, 25 cwt. 3 qrs. 17 lbs.; the second, 17 cwt. 2 qrs. 21 lbs. What is the weight of both?

5. A man set out for market with a load of hay weighing 36 cwt. 2 qrs. 15 lbs. net weight. He lost

a part of it; the remainder weighed 25 cwt. 1 qr. 8 lbs. How much did he lose?

6. If there are 196 lbs. in a barrel of flour, how many lbs. net weight are there in 10 barrels?

196 lbs. are 7 quarters gross. How many cwt. gross are there in 10 barrels of flour?

7. How many pounds are there in 100 oz.? In 650 oz.?

8. A barrel of flour weighs 7 quarters gross. How many tons gross are there in 100 barrels of flour?

9. What will be the expense of transporting by railroad 100 barrels of flour, 100 miles, at the rate of 3 dollars a ton?

What will be the expense of transporting a single barrel?

100 barrels are 700 qrs. gross weight. 400 qrs. = 100 cwt. = 5 tons; 300 qrs. = 75 cwt. = 3 tons 15 cwt.; this, added to 5 tons, makes 8 tons 15 cwt.

10. The freight of goods by wagon is about 20 dollars a ton gross for 100 miles. At this rate, what will be the cost of carrying a barrel of flour 100 miles?

TABLE OF TROY WEIGHT.

24 grains (gr.)	make	1 pennyweight,	marked dwt.
20 pennyweights,		1 ounce,	oz.
12 ounces,		1 pound,	lb.

This is used for weighing gold and silver. The pound Troy is nearly one fifth less than the pound Avoirdupois.

1. How many grains in 6 pennyweights? In 8 pennyweights? In 12 pennyweights? In 1 oz.? In 2 oz.? In 4 oz.? In 6 oz.?

2. How many pennyweights in 8 oz.? In 11 oz.? In 1 lb.? In 3 lbs.? In 8 lbs.? In 5 lbs.? In 1 lb. 3 oz.? In 2 lbs. 5 oz.?

3. How many oz. in 120 dwt.? In 480 dwt.? In 960 grs.? How many lbs. in 100 oz.? In 860 dwt.? In 1200 dwt.?

TABLE OF APOTHECARIES' WEIGHT.

20 grs.	make	1 scruple,	marked	℈
3 scruples,		1 dram,		ʒ
8 drams,		1 ounce,		℥
12 ounces,		1 pound,		℔

This table is used only by apothecaries in mixing medicines. The pound and ounce are the same as in Troy weight.

TABLE OF CLOTH MEASURE.

2¼ inches (in.)	make	1 nail,	marked	na.
4 nails,		1 quarter,		qr.
4 quarters,		1 yard,		yd.
3 quarters,		1 ell Flemish,		Fl. e.
5 quarters,		1 ell English,		E. e.
6 quarters,		1 ell French,		Fr. e.

1. How many inches in 1 qr.? In 1 yd.? In 3 yds.? In 1 ell Eng.? In 1 ell Fr.? In 1 ell Fl.?

2. How many inches in 4 yds.? In 7 yds.? In 12 yds.? In 10 yds.? In 20 yds.? In 6 yds. 3 qrs.? In 4 yds. 1 qr.?

TABLE OF WINE MEASURE.

4	gills (gi.)	make	1 pint,	marked	pt.
2	pints,		1 quart,		qt.
4	quarts,		1 gallon,		gal.
31½	gallons,		1 barrel,		bl.
63	gallons,		1 hogshead,		hhd.
2	hogsheads,		1 pipe,		p.
2	pipes,		1 tun,		T.

This table is used for measuring wine, spirits, cider, and water.

1. How many gills in 1 quart? In 1 gal.? In 4 gals.? In 6 gals.? In 10 gals.? In 13 gals.? In 15 gals.?

2. How many pints in 1 gal.? In 4 gals.? In 20 gals.? How many qts. in 1 barrel? In 1 hogshead?

3. How many gallons in 5 barrels? In 8 barrels? How many gals. in half a barrel? In one fourth of a barrel?

4. In 100 gals. how many barrels? In 300 gals. how many bls.?

5. At 14 cents a gallon, what is 1 qt. of vinegar worth? 3 qts.? 6 qts.? 10 qts.? 15 qts.? 21 qts.? 30 qts.?

6. What is 1 barrel of vinegar worth at 15 cts. a gallon? How much, if the price is 20 cts. a gal.?

TABLE OF ALE OR BEER MEASURE.

(Used in measuring malt liquors and milk.)

2	pints (pt.)	make	1 quart,	marked	qt.
4	quarts,		1 gallon,		gal.
36	gallons,		1 barrel,		bl.

The beer gallon is a little more than one fifth larger

than the wine gallon. There are other measures of beer besides those in the tables; as, the firkin, of 9 gallons; the kilderkin, 18; the hogshead, 54; but these are not much used in this country. A barrel of wine contains not quite three fourths as much as a barrel of beer.

1. In 1 bl. how many pints? How many pints in 3 bls.? How many gallons in 5 bls.? In 12 bls.? In 15 bls.? In 21 bls.?

2. In 100 gallons how many bls.? How many bls. in 400 gals.? First consider how many bls. there are in 360 gals.

MEASURE OF THE CIRCLE.

Every circle is supposed to have its circumference divided into 360 equal parts, called *degrees;* and each degree into 60 parts, called *minutes;* and each minute into 60 parts, called *seconds.* Whether the circle is great or small, it is still divided into 360 degrees. A degree, therefore, is always the same fixed part of the circumference of a circle, although its actual length is longer or shorter, according as the circle is great or small. The line passing from the centre to the circumference is called the *radius* of the circle. To give you some idea of the length of a degree in circles of different magnitudes, I will state that, on comparing a degree in any circle with its radius, it has been found to be about one fifty-eighth part of it. In other words, 58 degrees on the circumference of a circle are about equal to the radius. If a degree is 1 inch, the radius of that circle is 58 inches. If the radius of a carriage wheel is 29 inches, a degree on the rim of the same wheel will be half an inch.

If we take for illustration one of the largest-sized water-wheels, 29 feet in diameter, a degree on its rim would measure only 3 inches.

You may enlarge the circle in your mind, till you suppose it extending over a plain, with a radius of 58 rods. A degree on such a circle will measure 1 rod. If the radius is 58 miles, a degree will measure 1 mile. Now, the circle round the earth is so great in extent that a degree measures 69½ miles. This may aid you in forming a conception of the vast magnitude of the earth.

Each of these degrees is divided into 60 minutes, or geographical miles. A geographical mile, therefore, is about one sixth greater than a common mile. The Table of Circular Measure is as follows:

60 seconds (″). . . make. . . 1 minute, . . marked . ′.
60 minutes, (or geog. miles,) 1 degree, ... °.
360 degrees . . . . a circle.

The term *miles*, instead of *minutes*, can be used only in reference to the great circle of the earth.

As the earth turns round on its axis once in 24 hours, every place upon it passes in that time through the 360 degrees of its circle; and on the equator, which is the great circle, each of these degrees, we have seen, is 69½ miles.

How swiftly, then, does a body lying on the equator move in consequence of the daily revolution of the earth?

In 24 hours, it passes through 360 degrees; in one hour, then, it will pass through one twenty-fourth part as many, which is 15 degrees. If it pass through 15 degrees in one hour, how many minutes will it be in passing through 1 degree? One fifteenth of 60 minutes is 4 minutes. If it pass through a degree in 4 minutes, what part of a degree will it pass through in 1 minute? One fourth of a degree, or 15 geographical miles. If it pass through 15 geographical miles in 1 minute, in how many seconds will it pass

through one geographical mile? In 4 seconds; and in 1 second it will pass through one fourth of a geographical mile.

Now, a geographical mile on the equator is, as we have seen, longer than a common mile. We will here suppose it no longer, but of the same length, and it appears that an object on the equator moves, as the vast earth whirls round on its axis, one quarter of a mile every second of time. Reflect now, that, while the surface of the earth moves with such amazing speed, so vast is its size, that it occupies an entire day and night in turning once round.

If, as above stated, the earth turns from west to east at the rate of 15 degrees in an hour, we can, by knowing the time of day in any place, ascertain what time it is at a place any particular number of degrees east or west of it. It is noon at any place when the meridian of that place passes under the sun.

1. When it is noon at Boston, what time is it at a place 15 degrees west of Boston? At a place 15 degrees east of Boston?

2. When it is 12 o'clock at Boston, what time is it at a place 1 degree west of Boston? At a place 1 degree east of Boston? At a place 2 degrees west of Boston? At a place 2 degrees east of Boston? 3 degrees east? 3 degrees west? 4 degrees east? 4 degrees west? 5 degrees east? 5 degrees west?

3. Indianapolis is 15 degrees west of Boston. When it is noon at Boston, what time is it at Indianapolis? When it is sunset at Boston, where will the sun be at Indianapolis?

4. Niagara Falls are 8 degrees west of Boston. When it is noon at Boston, what time is it at Niagara Falls? When it is 4 o'clock at Niagara Falls, what time is it at Boston?

5. Washington city is 6 degrees west of Boston. If you set your watch with the sun at Boston, and

then carry it to Washington, your watch keeping accurate time all the while, when you arrive at Washington, will it be too fast or too slow? and how much?

6. Two travellers met at a public house. When one of them said to the other, "Friend, are you travelling east or west?" "I am direct from home," said the other, "where my watch agrees exactly with the sun, but here I find it is 10 minutes too fast. Now, if you can tell which way I am travelling, you are welcome to know."

Had he travelled east, or west? and how far?

7. Boston is 71 degrees west of London. When it is noon at Boston, what time is it in London?

8. The English convicts are transported to Botany Bay, 150 degrees east of London. When it is noon at London, what time is it in Botany Bay?

9. English traders are settled on Columbia River, 120 degrees west from London. What time is it there when it is noon in London?

10. If a man is on the equator, which way must he travel, and how many geographical miles, to have the day 4 minutes longer than 24 hours? How far, to have the day 2 minutes longer? How far, to have it 1 minute longer? How far must he travel to have the day 1 minute and a half longer? Which way must he travel, and how far, to have the day 1 minute shorter? 2 minutes shorter? 5 minutes shorter?

11. Suppose two birds start from the same place on the equator, and fly, one east and the other west, at the rate of 60 geographical miles an hour, and at the end of the hour it is just sunset to the bird flying east; how high is the sun then at the place where the other bird is?

How high was the sun at the place of their starting, when they set out?

12. A shipmaster sails from New York for Europe,

and for three days it is so cloudy that he cannot see the sun. On the fourth day, he takes an observation of the sun at noon; and by his chronometer, which gives the New York time, it is half past eleven. How many degrees east from New York has he sailed?

In what longitude is he then, if New York is 74° 1′ west from Greenwich?

SECTION VII.

PRIME NUMBERS.

Numbers may be divided into two great classes. The first class comprises such numbers as cannot be formed by the multiplication of any two or more numbers together; as, 1, 2, 3, 5, 11, 17. These are called *prime* numbers. The other class may be formed by multiplying two or more numbers together, as 4, which is formed by multiplying 2 by 2; 6, which is equal to 2×3; 10, which is equal to 2×5, &c. These are called *composite* numbers. These may always be formed by multiplying two or more prime numbers together. Thus all numbers are either prime, or are formed by the multiplication of prime numbers together.

In separating numbers into their factors, care should be taken that the factors be all prime. Thus, in resolving 30 into its factors, we may say it is formed by multiplying 5 by 6; but this is not sufficient, for 6 is not prime; it is formed of the factors 2 and 3. The prime factors of 30, therefore, are 2, 3, and 5. We may say that 30 is formed of the factors 3 and 10; but here, again, the analysis is not complete, for 10 is not prime; it is composed of the factors 2 and 5. Thus

we are brought to the same three factors as before, namely, 2, 3, and 5.

The following table of numbers from 1 to 100 will show what of them are prime, and what are the prime factors of those which are composite. This table should be carefully studied and made perfectly familiar. The analysis of composite numbers into their prime factors lies at the foundation of some of the most important operations in numbers, and affords an insight into some of the most intricate rules of arithmetic.

1, prime.
2, prime.
3, prime.
$4=2\times2$.
5, prime.
$6=3\times2$.
7, prime.
$8=2\times2\times2$.
$9=3\times3$.
$10=2\times5$.
11, prime.
$12=2\times2\times3$.
13, prime.
$14=2\times7$.
$15=3\times5$.
$16=2\times2\times2\times2$.
17, prime.
$18=2\times3\times3$.
19, prime.
$20=2\times2\times5$.
$21=3\times7$.
$22=2\times11$.
23, prime.
$24=2\times2\times2\times3$.
$25=5\times5$.
$26=2\times13$.
$27=3\times3\times3$.
$28=2\times2\times7$.
29, prime.
$30=2\times3\times5$.
31, prime.
$32=2\times2\times2\times2\times2$.
$33=3\times11$.
$34=2\times17$.
$35=5\times7$.
$36=2\times2\times3\times3$.
37, prime.
$38=2\times19$.
$39=3\times13$.
$40=2\times2\times2\times5$.
41, prime.
$42=2\times3\times7$.
43, prime.
$44=2\times2\times11$.
$45=5\times3\times3$.
$46=2\times23$.
47, prime.
$48=2\times2\times2\times2\times3$.
$49=7\times7$.
$50=2\times5\times5$.
$51=3\times17$.
$52=2\times2\times13$.

53, prime.
$54=2\times3\times3\times3$.
$55=5\times11$.
$56=2\times2\times2\times7$.
$57=3\times19$.
$58=2\times29$.
59, prime.
$60=2\times2\times3\times5$.
61, prime.
$62=2\times31$.
$63=3\times3\times7$.
$64=2\times2\times2\times2\times2\times2$.
$65=5\times13$.
$66=2\times3\times11$.
67, prime.
$68=2\times2\times17$.
$69=3\times23$.
$70=2\times5\times7$.
71, prime.
$72=2\times2\times2\times3\times3$.
73, prime.
$74=2\times37$.
$75=3\times5\times5$.
$76=2\times2\times19$.
$77=7\times11$.
$78=2\times3\times13$.
79, prime.
$80=2\times2\times2\times2\times5$.
$81=3\times3\times3\times3$.
$82=2\times41$.
83, prime.
$84=2\times2\times3\times7$.
$85=5\times17$.
$86=2\times43$.
$87=3\times29$.
$88=2\times2\times2\times11$.
89, prime.
$90=2\times3\times3\times5$.
$91=7\times13$.
$92=2\times2\times23$.
$93=3\times31$.
$94=2\times47$.
$95=5\times19$.
$96=2\times2\times2\times2\times2\times3$.
97, prime.
$98=2\times7\times7$.
$99=3\times3\times11$.
$100=2\times2\times5\times5$.

On examining this table, several things may be observed.

1. All the even numbers are composite; for they are all divisible by 2. So it appears in the table, with the exception of the number 2, which is regarded as prime because it is divisible only by itself.

2. Several of the numbers given above are powers of their prime factors. Thus 4 is the 2d power of 2, 8 the third power of 2, 16 the 4th power of 2, 32 the 5th, 64 the 6th. 9, 27, and 81, are the 2d, 3d, and 4th powers of 3. 25 is the 2d power of 5, 49 the 2d power of 7.

3. If you double the number of times a factor is taken, you obtain the square of the number they at first made. Thus 4 is obtained by taking 2 twice as a factor. If you take it twice as many times, that is, 4 times, as a factor, you obtain 16, which is the square of 4.

9 is obtained by taking 3 twice as a factor. If you double the number of times it is taken, thus, $3\times3\times3\times3$, you obtain the square of 9.

8 is obtained by taking 2 three times as a factor. If you take it 6 times, you obtain 64, the square of 8.

So universally, if you double the number of times a factor is taken to produce a certain number, you obtain, not twice that number, but the square of it.

I will make a single remark here about the prime numbers, and then call your attention to the composite numbers.

Since the prime numbers are not formed by multiplying any two or more numbers together, they cannot be divided by any number. You will observe, however, that any number whatever may be divided by itself, and may also be divided by 1; but 1 is a unit, and not a number; and by dividing a number by itself, or by 1, you obtain no new number. Dividing the number by itself, you obtain 1; and dividing by 1, you obtain the number itself. Such an operation, therefore, brings out nothing new. It is only another way of expressing what was just as plain before. In the same way, we may sometimes regard a number as produced by multiplying itself into 1; thus, $7=7\times1$; but this is not multiplication, but only an expression in the form of multiplication. It produces no new number, and is employed only for convenience, in order to make the reasoning more plain.

Composite numbers can be divided by their factors. Thus you can divide 10 either by 2 or by 5, and by no other number. If you divide by 2, you obtain 5

for the answer, or quotient; if you divide by 5, you obtain 2 for the answer. Dividing by a number, then, is the same as erasing that number as a factor, and will always give for the answer the other factor, or factors. Thus, dividing 10 by 2 you may represent thus, $\cancel{2}\times5$, leaving the factor 5 for the answer; dividing 10 by 5, thus, $2\times\cancel{5}$, leaving 2 for the answer. Divide 21 by 3, thus, $\cancel{3}\times7$. Divide 12 by 3, thus, $\cancel{4}\times3$, or $\cancel{2}\times\cancel{2}\times3$.

It is plain, therefore, that if you express any number by its factors, you can at once see what numbers you can divide it by. You can divide it by each of its prime factors, or by any combination of them, and by no other number. Thus $6=2\times3$ you can divide by 2 or by 3; $8=2\times2\times2$ you can divide by 2, and that quotient by 2, and that by 2 again; $30=2\times3\times5$ you can divide by 2, or 3, or 5, or by any two of them combined.

Any composite number may be divided by any of its prime factors, or by any combination of them.

By what numbers can you divide 15? 18? 20? 21? 26? 27? 36? 42? 46? 48? 49? 50?

Sometimes we have two numbers, and we wish to know if there is any number that will divide them both. This we can ascertain if we express each number by means of its prime factors, and then see if the same factor is found in both. If so, they are both divisible by that number. Thus, if we wish to know whether any number will divide both 9 and 15, we express them thus, 3×3 and 3×5. Now, 3 appears as a factor in both; they can both, therefore, be divided by 3. This number, 3, is called the *common divisor*, because it is a divisor common to several numbers. If we wish to know whether any number will divide both 15 and 8, we express 15 by its factors, 5×3; and 8 by its factors, $2\times2\times2$. Now, there is no factor common to both; no number, therefore, will

divide them both; in other words, they have no common divisor. Numbers which have no common divisor are said to be prime to each other. They may be composite considered by themselves, as is the case with 8 and 15; but if they have no common divisor, they are said to be prime to each other. Numbers which have a common divisor are said to be composite to each other. If there are more than two numbers, they must be treated in the same way. Each must be written in the form of its prime factors; and then, if any one number appears as a factor in them all, they are divisible by that.

Is there any common divisor to 9, 14, and 27? Written in the form of their factors, they stand thus, 2×3; 2×7; $3\times3\times3$. They have, therefore, no common divisor; for, though 3 or 9 will divide both the first and the third number, it will not divide the second; and neither 2 nor 7, which are the factors of the second number, appear in the first or third. 9, 14, and 27, are, therefore, prime to each other.

What is the common divisor of 15 and 27? Of 14 and 22? Of 21 and 49? Of 35 and 28? Of 6 and 21?

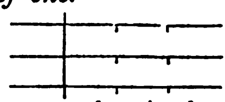
Let us now take the following question: What is the common divisor of 18 and 30? By inspecting their factors, $2\times3\times3$, and $2\times3\times5$, we find that 2×3, or 6, is common to both; 6 is therefore the greatest common divisor.

What is the greatest common divisor of 18 and 27? Of 4, 8, and 36? Of 15 and 45? Of 27 and 45? Of 40, 64, and 16? Of 44 and 24? Of 75 and 15? Of 80 and 100? Of 60 and 24? Of 35, 21, and 49? Of 15 and 50?

We have seen that a composite number can be divided only by its factors, and that prime numbers cannot be divided at all. It is frequently necessary, however, to attempt the division of prime numbers, and to divide composite nnmbers by some number different

from their factors. For example, we may wish to divide 9 by 4, or to obtain one fourth of 9. Now, 4 is not a factor of 9, and the actual division of 9 by 4 is, strictly speaking, impossible. We proceed in this way. We divide 8 by 4, and obtain 2 for the answer, and we have a remainder of 1, which we have not divided. To show that we design this to be divided by 4, we write the 4 under it, with a line between, thus, $\frac{1}{4}$. In this way we indicate plainly enough what the answer is, although we have no one figure that will express it.

Again, let us divide 15 by 4. We divide 12 by 4, obtaining for the quotient 3, and we have 3 remaining. This we cannot divide by 4 so as to express the quotient by a single figure. We therefore indicate the division, without performing it, by writing the divisor, 4, under the dividend, 3, thus, $\frac{3}{4}$. When we divide a number by 4, we obtain one fourth of it. The expression $\frac{3}{4}$, therefore, signifies *one fourth of three.* But one fourth of three is the same quantity as *three fourths of one.*

Thus  the perpendicular stroke cuts off, on the left, one fourth part of the three whole lines. By looking at what is thus cut off, you see that it is just equal to the *three fourths* of one whole line, on the right hand of the perpendicular mark. We may, therefore, call $\frac{3}{4}$ one fourth of three, or three fourths of one; or, simply, *three fourths*, meaning *three fourths of one;* and, as this is the shorter expression, it is the one usually employed.

Questions.

1. What will be the remainder in dividing 18 by 5? How will you express this part of the division, which

it is impossible to perform? In what different ways can you read the expression?

2. How will you draw lines on the board, or slate, so as to show that one fifth of three is equal to three fifths of one?

3. What will be the remainder in dividing 23 by 6? How will you express the part of the division which is not performed? In what different ways can you read the expression?

4. How will you draw lines, so as to show that one sixth of five is equal to five sixths of one?

5. In all the above cases, which is the larger, the dividend, or the divisor?

6. In all the above expressions, what is the value of the quantity expressed, compared to one? Is it greater, or less, than one?

In the same way as in the above cases, we may denote the division of any number by a number greater than itself; by writing the divisor under the dividend; as, $\frac{3}{7}$; $\frac{5}{8}$; $\frac{13}{14}$. These expressions are called *Fractions.* A fraction, therefore, is an expression denoting the division of a number by a number greater than itself; and as the quantity signified by such an expression is necessarily less than *one,* we may say, more briefly,

A fraction is an expression for a quantity less than one.

It is expressed by means of two numbers,—the smaller (the dividend) written above, the larger (the divisor) written below, a horizontal line.

The number below the line is called the *denominator,* because it shows into *how many* equal parts the number, or unit, is divided; the number above the line is called the *numerator,* because it shows how many units are divided, or how many parts of a divided unit are taken.

7. In the expressions $\frac{3}{7}$, $\frac{4}{9}$, $\frac{5}{11}$, is the quantity denoted in each expression greater than 1, or less? and why?

8. In the expressions $\frac{9}{7}$, $\frac{4}{3}$, $\frac{8}{5}$, is the quantity denoted in each case greater than 1, or less? and why?

It is sometimes convenient to express quantities which are not less than 1, in the form of fractions; as, $\frac{8}{8}$, $\frac{5}{3}$, &c. These are called *Improper Fractions*, while the others are called, in distinction from these, *Proper Fractions*. An improper fraction may always be reduced to a whole number and a proper fraction.

A whole number and a fraction taken together, is called a *Mixed Number*.

9. Reduce to a mixed number, $\frac{13}{8}$; $\frac{15}{7}$; $\frac{22}{13}$.

10. Reduce to an improper fraction, $4\frac{1}{5}$; $7\frac{1}{4}$; $9\frac{2}{3}$.

Questions.

What is meant by a common divisor?
What is meant by the greatest common divisor?
When are numbers prime to each other?
When are numbers composite to each other?
What is the process of dividing 13 by 4?
In dividing 16 by 5? In dividing 25 by 6?

What are Fractions?

Explain what is signified by each of the numbers in the fraction $\frac{2}{3}$. In $\frac{4}{7}$. In $\frac{6}{11}$. In $\frac{5}{12}$. In $\frac{3}{4}$. In $\frac{15}{16}$.

A man bought a barrel of flour, and gave away two fifths of it. What fraction will express what he gave away? What fraction will express what he kept?

A man bought a load of hay, and sold two elevenths of it. What fraction will express what he sold? What fraction will express what he kept?

What is a proper fraction? Give an example.
What is an improper fraction? Give an example.
When is the value of a fraction just equal to 1?

SECTION VIII.

MULTIPLICATION AND DIVISION OF FRACTIONS.

We have seen that a fraction is not a simple expression, but composed of two numbers; and its value cannot be determined by one of these numbers alone, but by both taken in connection. By looking at the numerator, you cannot tell the value of the fraction, unless you know what the denominator is. By looking at the denominator, you cannot tell the value of the fraction, unless you know what the numerator is.

Let us now observe the effect of altering one of the terms of the fraction without altering the other. We will take the fraction $\frac{2}{5}$. If we increase the numerator by 1, making it $\frac{3}{5}$, we increase the value of the fraction, for we take one fifth more than we had before. So, if we multiply the numerator by 2, making it $\frac{4}{5}$, we double the value of the fraction; and so of any other numbers, if we multiply the numerator, we multiply the value of the fraction. And, by the same reasoning, if we divide the numerator by 2, we divide the fraction by 2, for $\frac{1}{5}$ is plainly one half as great as $\frac{2}{5}$. So of all other numbers, by dividing the numerator we divide the fraction.

Let us now observe the effect of altering the denominator. If we increase the denominator of the fraction $\frac{2}{5}$ by 1, making it $\frac{2}{6}$, we have not increased the fraction, but diminished it; for one sixth is less than one fifth, and any number of sixths are less than the same number of fifths. We will multiply the denominator of the fraction $\frac{2}{5}$ by 2, making $\frac{2}{10}$. What effect has been produced on the value of the fraction? One tenth is half as great as one fifth, and two tenths are half as great as two fifths. The fraction is, therefore, half

as great as it was before; that is, it has been divided by 2. Multiplying the denominator, therefore, divides the value of the fraction.

We will now divide the denominator. Take the fraction $\frac{3}{8}$; dividing the denominator by 2, we have $\frac{3}{4}$. Now, as this is twice as great as $\frac{3}{8}$, we have multiplied the fraction, by dividing the denominator.

There are, then, two ways of multiplying a fraction. We may multiply the numerator; or, if the multiplier is a factor of the denominator, we may divide the denominator. Thus, to multiply $\frac{3}{8}$ by 2, we may multiply the numerator, which gives $\frac{6}{8}$, or divide the denominator, which gives $\frac{3}{4}$, equal to $\frac{6}{8}$.

To divide a fraction, we may either divide the numerator, if the divisor is a factor of it, or we may multiply the denominator. Thus, to divide $\frac{6}{7}$ by 3, we may divide the numerator, giving $\frac{2}{7}$, or we may multiply the denominator, which gives $\frac{6}{21}$, which is equal to $\frac{2}{7}$.

We will now multiply both terms of the fraction by the same number. Multiplying both terms of the fraction $\frac{2}{3}$ by 3, we have $\frac{6}{9}$. Here the denominator, expressing the number of parts into which the unit is divided, is three times as great as it was before; consequently each of the parts is only one third as great; but the numerator has also been multiplied by 3, so that three times as many parts are taken, and this makes the value of the fraction just equal to what it was before. So we may multiply, by any number whatever, both terms of the fraction $\frac{2}{3}$, and the value will still be the same as before; for example, $\frac{4}{6}$, $\frac{6}{9}$, $\frac{8}{12}$, $\frac{10}{15}$, $\frac{12}{18}$, each of which is equal to $\frac{2}{3}$. We may, then, at any time multiply both terms of a fraction by the same number, without altering the value of the fraction. By the same reasoning, we may divide both terms of a fraction by the same number without altering its value. Taking the examples above, we may divide

the terms of $\frac{4}{6}$ by 2, and we obtain $\frac{2}{3}$; dividing the terms of $\frac{6}{9}$ by 3 gives us $\frac{2}{3}$, and so of the others. $\frac{2}{3}$ is the same fraction as $\frac{4}{6}$, $\frac{6}{9}$, $\frac{8}{12}$, &c., but it is expressed in lower terms, and therefore is more convenient. It is easier to write $\frac{1}{2}$ than it is to write $\frac{16}{32}$, though both have the same value.

To reduce a fraction to its lowest terms, we divide both the numerator and denominator by their greatest common divisor. To find the greatest common divisor, separate each term into its prime factors, and erase those which are common to both. The remaining factors will express the value of the fraction in its lowest terms.

Treating the above fractions in this way, they appear thus,

$$\frac{4}{6}=\frac{\cancel{2}\times 2}{\cancel{2}\times 3},\ \frac{6}{9}=\frac{2\times\cancel{3}}{3\times\cancel{3}},\ \frac{8}{12}=\frac{\cancel{2}\times\cancel{2}\times 2}{\cancel{2}\times\cancel{2}\times 3},\ \frac{10}{15}=\frac{2\times\cancel{5}}{3\times\cancel{5}},\ \frac{12}{18}=\frac{\cancel{2}\times 2\times\cancel{3}}{\cancel{2}\times 3\times\cancel{3}},$$

leaving, in each case, $\frac{2}{3}$.

In how many ways can you obtain the answer to the following questions? $\frac{3}{4}\times 2$? $\frac{5}{6}\times 3$? $\frac{3}{8}\times 4$? $\frac{5}{12}\times 2$?

In how many ways can you obtain the answer to the following? $\frac{2}{7}\times 3$? $\frac{4}{9}\times 2$? $\frac{3}{5}\times 4$? $\frac{2}{11}\times 6$? $\frac{5}{12}\times 5$? $\frac{3}{14}\times 3$?

In how many ways can you obtain the answer to the following? $\frac{6}{7}\div 3$? $\frac{8}{9}\div 4$? $\frac{9}{10}\div 3$? $\frac{14}{15}\div 2$? $\frac{12}{17}\div 4$?

In how many ways can you obtain an answer to the following? $\frac{5}{8}\div 2$? $\frac{6}{7}\div 4$? $\frac{7}{8}\div 3$? $\frac{9}{10}\div 2$? $\frac{12}{13}\div 5$?

Reduce to their lowest terms each of the following fractions: $\frac{6}{8}$; $\frac{14}{21}$; $\frac{3}{33}$; $\frac{22}{32}$; $\frac{40}{60}$; $\frac{15}{50}$; $\frac{18}{28}$; $\frac{24}{27}$; $\frac{26}{38}$; $\frac{6}{16}$; $\frac{9}{15}$; $\frac{15}{33}$; $\frac{8}{26}$; $\frac{30}{48}$; $\frac{72}{95}$.

TO FIND THE DIVISORS OF NUMBERS.

Reduce the fraction $\frac{117}{153}$ to its lowest terms.

You will not see immediately that these two num-

bers have any common divisor. To assist you to reduce fractions of this kind, something will here be said about the way of finding the divisors of numbers. Let us first inquire what numbers can be divided by 2.

We have seen that all even numbers, and only those, can be divided by 2.

What numbers can be divided by 4?

If you examine, you will find that all even tens are divisible by 4; as, 20, 40, 60, &c. If, therefore, the tens are even, and the units are divisible by 4, then the whole is divisible by 4. But the only unit numbers divisible by 4 are 4 and 8; therefore, if the tens are even, and the unit number is 4 or 8, the whole is divisible by 4; as, 84, 88; 124, 128; 148, 364; &c.

Again; as 10, when divided by 4, leaves a remainder of 2, any odd number of tens will do the same; as, 30, 50, 70, 90; for every odd number of tens is an even number of tens $+10$. If, then, the number of tens is odd, the units must be two less than 4 or 8, in order to be divisible by 4; that is, if the tens are odd, and the units 2 or 6, the whole is divisible by 4; as, 72, 96, 52, &c.

Are the following even numbers divisible by 4, or only by 2? and why? 126; 82; 94; 92; 138; 156; 346; 548; 76; 58; 392.

What numbers can be divided by 8?

As 100 divided by 8 leaves a remainder of 4, ($8\times12=96$,) it follows that 200 will be exactly divisible by 8, for the two remainders of 4 will make 8. If 200 is divisible by 8, it follows that all even hundreds are divisible by 8; as, 400, 600, 1400, &c.

If, therefore, the hundreds are even, and the tens and units are divisible by 8, the whole number will be divisible by 8; as, 248, 672, 1456, &c.

Again; if the hundreds are odd, and the tens and units are 4 less than some multiple of 8, the whole

number will be divisible by 8; for the odd hundred. divided by 8, leaves a remainder of 4; and this, added to the tens and units, will make an exact multiple of 8.

Are the following numbers divisible by 8, or by 4? and why? 444; 944; 136; 1328; 712; 532; 816; 516; 384; 128; 1236.

What numbers are divisible by 5? All tens are divisible by 5; consequently, if the unit figure is 5 or 0, the whole number is divisible by 5.

What numbers are divisible by 3? By examining the multiples of 3, we shall find this singular fact, that the sum of the figures which express any multiple of 3 is itself a multiple of 3. Take the multiples of 3 from 12 to 24; 12, 15, 18, 21, 24; by adding the figures which express any one of these multiples, we find that the sum is a multiple of 3. The figures of 12 added are $1+2=3$, of 15 are $1+5=6$, of 18 are $1+8=9$, of 21 are $2+1=3$, of 24 are $2+4=6$. The same is true of all multiples of 3.

It will also be found, that, if you add the figures of any number, and the sum is a multiple of 3, the whole number is a multiple of 3. To know, then, if a number is a multiple of 3, add together the figures that express the number, and if the sum is a multiple of 3, the whole number is a multiple of 3.

Are the following numbers divisible by 3? 471; 59; 115; 642; 624; 138; 234; 742; 894.

It follows from what has been said, that, if any number is divisible by 3, any other number expressed by the same figures differently arranged will also be divisible by 3; for the sum made by adding the figures will be the *same* in whatever order they are taken.

Thus, if 936 is divisible by 3; 369, 396, 963, 639, 693, are each divisible by 3.

We will next inquire what numbers are divisible by 6. As $6=2\times3$, any number that is divisible by

2 and by 3 is divisible by 6. You have learned what numbers are divisible by 3, and what by 2. If a number combines both these conditions, it is divisible by 6; that is, all numbers are divisible by 6, the sum of whose figures is a multiple of 3, and whose last figure is an even number.

What combinations of the figures 1, 2, 3, will give numbers divisible by 6? and what by 3 only?

Next let us inquire what numbers are divisible by 9.

If the figures which express any multiple of 9, as 18, 27, 36, 45, 54, be added together, the sum will be a multiple of 9.

Also, if the figures of any number be added together, and the sum is a multiple of 9, the whole number is divisible by 9.

Are the following numbers divisible by 9? and why? 936; 972; 396; 423; 387; 527; 441; 416; 315; 756.

Any number divisible by 9 and by 2 is divisible by 9×2, or 18. Which of the above numbers are divisible by 18?

Any number divisible by 9 and by 4 is divisible by 9×4, or 36. Which of the above numbers are divisible by 36?

Any number divisible by 9 and by 8 is divisible by 9×8, or 72. Is either of the above numbers divisible by 72?

Any number divisible by 9 and by 5 is divisible by 9×5, or 45. Which of the above numbers is divisible by 45?

What are divisors of 124? Of 176? Of 252? Of 384? Of 153? Of 186? Of 207? Of 702? Of 4041?

We will now return to the fraction that was first given. Reduce $\frac{117}{153}$ to its lowest terms.

Reduce to lowest terms, $\frac{171}{189}$; $\frac{243}{324}$; $\frac{405}{486}$.

Reduce to lowest terms, $\frac{225}{765}$; $\frac{279}{315}$; $\frac{448}{576}$.

SECTION IX.

MULTIPLICATION OF FRACTIONS BY FRACTIONS.

We have seen how we may multiply or divide a fraction by a whole number. We will now inquire how we can multiply or divide one fraction by another. Let us multiply $\frac{3}{7}$ by $\frac{2}{5}$. First multiply $\frac{3}{7}$ by 2, which gives $\frac{6}{7}$ for the answer. But here we have multiplied by 2, instead of the real multiplier, $\frac{2}{5}$. Now, 2 is 5 times greater than $\frac{2}{5}$; the product $\frac{6}{7}$, then, is 5 times greater than it should be. It must therefore be divided by 5. We divide $\frac{6}{7}$ by 5 by multiplying the denominator by 5, giving $\frac{6}{35}$ for the answer.

In the same way multiply $\frac{5}{6}$ by $\frac{3}{11}$; $\frac{6}{7}\times\frac{4}{5}$; $\frac{5}{8}\times\frac{7}{9}$.

DIVISION OF FRACTIONS BY FRACTIONS.

Let us now divide $\frac{4}{5}$ by $\frac{3}{7}$. First divide $\frac{4}{5}$ by 3. This we do by multiplying the denominator by 3, giving for the answer $\frac{4}{15}$. Here, however, we have divided by 3, instead of the true divisor, $\frac{3}{7}$. We have used a divisor seven times too large. The quotient, therefore; will be seven times too small; $\frac{4}{15}$ must therefore be multiplied by 7, making the answer $\frac{28}{15}$. In the same way perform the following: $\frac{2}{3}\div\frac{3}{7}$; $\frac{5}{6}\div\frac{3}{4}$; $\frac{2}{7}\div\frac{3}{8}$; $\frac{5}{9}\div\frac{2}{7}$; $\frac{1}{5}\div\frac{1}{8}$.

The above analysis shows the grounds of the rules usually given in arithmetics for the multiplication and division of fractions.

For Multiplication, multiply the numerators together for a new numerator, and the denominators for a new denominator.

For Division, invert the divisor, and proceed as in multiplication.

Sometimes we wish to find the value of a com-

pound fraction, as $\frac{2}{3}$ of $\frac{3}{4}$. In such cases, we may understand the sign of multiplication, $\times$, to stand in the place of the word *of*, and treat it as a case of multiplication; for, in the above example, it is plain that one third of $\frac{3}{4}$ is $\frac{3}{12}$; and two thirds is twice as much, that is, $\frac{6}{12}$.

What is $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{9}$? Multiplying as we have done above, we have for the answer $\frac{24}{108}$. But this operation may be shortened. We see that 4 appears as a factor both in the numerator and the denominator. We may, then, cancel them both, which will have the same effect as dividing both terms of the answer by 4. Again; 3 appears in both the numerator and the denominator; for in the denominator it is a factor of 9. We may therefore cancel 3 in both terms.

The question will then appear thus, $\frac{2}{3}\times\frac{\cancel{3}}{\cancel{4}}\times\frac{\cancel{4}}{\cancel{9}3}$, substituting 3 in place of the 9. Multiplying together the terms that now remain, we have $\frac{2}{9}$ for the answer. This is the same fraction as $\frac{24}{108}$. If you separate the terms of $\frac{24}{108}$ into their prime factors, and cancel what are common to both, the remaining factors will give the fraction $\frac{2}{9}$.

Multiply the fractions $\frac{3}{8}\times\frac{2}{9}\times\frac{14}{15}\times\frac{5}{7}$. Writing the terms that are composite in the form of their prime factors, and cancelling factors that are common in both, it will stand $\frac{\cancel{3}}{\cancel{2}\times\cancel{2}\times2}\times\frac{\cancel{2}}{\cancel{3}\times3}\times\frac{\cancel{2}\times\cancel{7}}{3\times\cancel{5}}\times\frac{\cancel{5}}{\cancel{7}}$, which gives $\frac{1}{18}$.

Multiply $\frac{2}{9}\times\frac{3}{4}\times\frac{6}{7}$; $\frac{21}{24}\times\frac{12}{15}\times\frac{5}{6}$.
Multiply $\frac{20}{27}\times\frac{9}{10}$; $\frac{15}{16}\times\frac{8}{27}$; $\frac{18}{28}\times\frac{3}{6}\times\frac{4}{9}$.

TO MULTIPLY OR DIVIDE WHOLE NUMBERS BY FRACTIONS.

The above examples will show how to multiply or divide a whole number by a fraction.

Multiply 7 by $\frac{4}{5}$. Multiplying 7 by 4 gives 28, which is 5 times too great, because 4 is five times greater than $\frac{4}{5}$. We must therefore divide the answer by 5, thus, $\frac{28}{5}$. As this is more than 1, we can reduce it to a whole number and a fraction. As $\frac{5}{5}$ is equal to 1, $\frac{25}{5}$ will be equal to 5; $\frac{28}{5}$, therefore, is equal to $5\frac{3}{5}$.

In this way multiply 6 by $\frac{7}{8}$; 9 by $\frac{3}{4}$; 8 by $\frac{5}{7}$.

This operation is in fact the same as multiplying a fraction by a whole number, which has been treated of already.

Let us next divide 7 by $\frac{3}{4}$. Dividing 7 by 3, we have $\frac{7}{3}$. Here, however, we have divided by a number 4 times too great; for 3 is four times greater than $\frac{3}{4}$. If the divisor is 4 times too great, the quotient will be 4 times too small; $\frac{7}{3}$, therefore, must be multiplied by 4, giving $\frac{28}{3}$ for the answer.

Divide 8 by $\frac{5}{7}$; $9 \div \frac{2}{5}$; $11 \div \frac{3}{7}$; $10 \div \frac{3}{11}$.

To reduce an improper fraction, as $\frac{13}{4}$, to a whole number and a proper fraction, we have only to consider how many whole ones the fraction is equal to, and how much remains. Thus, $\frac{12}{4}$ is equal to 3; $\frac{13}{4}$, therefore, is equal to $3\frac{1}{4}$.

Reduce $\frac{8}{3}$; $\frac{17}{5}$; $\frac{12}{7}$; $\frac{36}{5}$; $\frac{42}{8}$; $\frac{56}{9}$; $\frac{31}{14}$; $\frac{91}{12}$; $\frac{87}{5}$.

In like manner, if we have a whole number and a fraction, we may always reduce it to an improper fraction.

ADDITION AND SUBTRACTION OF FRACTIONS.

Suppose we wish to add together $3\frac{1}{2}$, so that its value shall be expressed in a single expression; we must change 3 to halves, which will be $\frac{6}{2}$; adding $\frac{1}{2}$ to this, we have $\frac{7}{2}$ for the answer.

In order to unite separate numbers into one expression, they must be of the same kind. We cannot unite 2 bushels and 3 pecks in one expression. It is still 2

bushels and 3 pecks, and we can make nothing else of it. But if we change the bushels to pecks, making 8 pecks, we can then add the 3 pecks, and bring it all into one expression, 11 pecks. So, to unite $5\frac{2}{3}$, we must change the 5 to thirds, making $\frac{15}{3}$, and add the $\frac{2}{3}$, making $\frac{17}{3}$. This is called reducing a mixed number to an improper fraction.

Reduce to an improper fraction, $7\frac{1}{3}$; $8\frac{1}{4}$; $4\frac{3}{7}$; $5\frac{1}{2}$; $6\frac{1}{4}$; $9\frac{1}{2}$; $3\frac{2}{3}$; $5\frac{3}{7}$; $15\frac{1}{2}$; $16\frac{3}{4}$; $13\frac{2}{3}$; $20\frac{3}{4}$; $21\frac{1}{5}$.

Supposing we wish to add $\frac{1}{2}$ to $\frac{1}{4}$, we must change the $\frac{1}{2}$ to fourths, making $\frac{2}{4}$; adding these, we have $\frac{3}{4}$ for the answer.

Add $\frac{1}{2}$ to $\frac{1}{12}$. $\frac{1}{2}=\frac{6}{12}$; $\frac{6}{12}+\frac{1}{12}=\frac{7}{12}$, *Ans.*

Add $\frac{1}{7}$ to $\frac{3}{14}$; $\frac{1}{3}+\frac{4}{15}$; $\frac{1}{5}+\frac{4}{20}$; $\frac{4}{7}+\frac{3}{28}$; $\frac{5}{6}+\frac{17}{18}$.

Let us now add $\frac{2}{3}$ and $\frac{4}{5}$. This question, you perceive, has a difficulty which the former ones had not; for $\frac{2}{3}$ is no number of fifths, and therefore we cannot bring the fraction into fifths by any multiplication. We want a number for the denominator which can be divided both by 3 and by 5. Now, if you examine, you will find no such number until you come to 15. This is, of course, divisible by 3 and by 5, for these are its factors. We will then take 15 for the denominator. This we call the *common denominator*. Taking, now, the fractions $\frac{2}{3}$ and $\frac{4}{5}$, and changing the denominator, 3, to 15, we see that we have made it 5 times as large as it was before; that is, we have multiplied it by 5. We must therefore multiply the numerator by 5, to preserve the value of the fraction. The fraction $\frac{2}{3}$ then becomes $\frac{10}{15}$, without altering its value. Passing, now, to the second fraction, $\frac{4}{5}$, we see that, in changing the denominator to 15, we have multiplied it by 3; we must therefore multiply its numerator by 3. This will make the fraction $\frac{12}{15}$. The two fractions will stand, then, $\frac{10}{15}+\frac{12}{15}$, which added together are $\frac{22}{15}=1\frac{7}{15}$.

TO FIND A COMMON DENOMINATOR.

We can always obtain a common denominator, by multiplying all the denominators together. Then, for the numerators, consider, in the case of each fraction, what its denominator has been multiplied by, in order to change it to the common denominator, and multiply the numerator by the same number. Thus each fraction will have had its numerator and its denominator multiplied by the same number, and so its value will not be changed.

What is the value of $\frac{1}{2}+\frac{4}{7}$? Of $\frac{2}{3}+\frac{3}{4}$? Of $\frac{6}{7}+\frac{4}{5}$? Of $\frac{1}{2}+\frac{2}{3}$? Of $\frac{1}{5}+\frac{2}{9}$? Of $\frac{6}{7}+\frac{2}{3}$? Of $\frac{3}{7}+\frac{2}{8}$? Of $\frac{8}{9}+\frac{3}{4}$?

Supposing we wish to add the fractions $\frac{3}{4}$ and $\frac{5}{6}$; we can proceed as above, and, with the common denominator, 24, the fractions will be $\frac{18}{24}+\frac{20}{24}$. But we need not employ so large a denominator as 24. We seek the smallest denominator that shall contain both 4 and 6 as a factor. If, now, we separate 4 and 6 into their prime factors, we shall find the factor 2 belonging both to 4 and to 6; thus, 2×2, 2×3. Now, one of these may be cancelled, and we shall still have 2×2 for the number 4, and 2×3 for the number 6. Multiplying the factors which remain, $2\times2\times3$, we have 12 for the smallest common denominator.

From this we see, that, when both the denominators contain the same factor, we may reject it from one of them, and multiply together the factors that remain.

Add $\frac{3}{8}$ to $\frac{5}{12}$. Here 2×2 is common to both denominators. Rejecting it in one, and multiplying, we obtain 24 for the least common denominator.

Add $\frac{5}{18}$ to $\frac{6}{27}$. Here 3×3 is common to both denominators. Rejecting it in one, and multiplying what remains, we have 54 for the least common denominator.

Add $\frac{17}{20}$ to $\frac{5}{32}$. Add $\frac{3}{8}$ to $\frac{1}{36}$. Add $\frac{4}{15}$ to $\frac{2}{25}$.

When more fractions than two are to be added, it is often most convenient to add two together first, and then add a third to the sum of these, and so on.

Add $\frac{2}{3}+\frac{1}{6}+\frac{3}{4}$. First add $\frac{2}{3}$ and $\frac{1}{6}$, which equal $\frac{5}{6}$. Next, $\frac{5}{6}+\frac{3}{4}$; $\frac{5}{6}=\frac{10}{12}$, and $\frac{3}{4}=\frac{9}{12}$; $\frac{10}{12}+\frac{9}{12}=\frac{19}{12}=1\frac{7}{12}$, *Ans.*

Add $\frac{1}{5}+\frac{2}{3}+\frac{3}{10}$. First add $\frac{1}{5}$ and $\frac{3}{10}$; then to the sum of these add $\frac{2}{3}$.

Add $\frac{2}{9}+\frac{1}{12}+\frac{5}{6}$. Add $\frac{3}{5}+\frac{2}{3}+\frac{4}{15}$. Add $\frac{1}{7}+\frac{3}{4}+\frac{3}{8}$.

Add $\frac{3}{13}+\frac{7}{9}$. Add $\frac{8}{11}+\frac{3}{7}$. Add $\frac{7}{12}+\frac{5}{24}+\frac{1}{10}$.

From $\frac{11}{12}$ subtract $\frac{5}{12}$. From $\frac{5}{6}$ sub. $\frac{7}{12}$. From $\frac{11}{16}$ sub. $\frac{5}{32}$.

From $\frac{18}{19}$ sub. $\frac{3}{4}$. From $\frac{20}{23}$ sub. $\frac{7}{10}$. From $\frac{31}{35}$ sub. $\frac{9}{12}$.

Miscellaneous Examples.

1. A man spends $\frac{1}{7}$ of a dollar in a day. What part of a dollar will he spend in 5 days? How much will he spend in 9 days? How much in 11 days?

2. A man earns $\frac{8}{9}$ of a dollar in a day. How much will he earn in half a day? How much in $\frac{1}{4}$ of a day? How much in $\frac{1}{8}$ of a day?

Here consider whether you can divide the numerator.

3. A man earns $\frac{7}{8}$ of a dollar in a day. How much can he earn in half a day? How much in $\frac{1}{4}$ of a day? How much in $\frac{1}{8}$ of a day?

Consider whether you can divide the numerator, and, if you cannot, what you must do.

4. A vessel filled with water leaks so that $\frac{3}{8}$ of its contents will leak out in a week. At this rate, what part will leak out in a day?

What is $\frac{1}{7}$ of $\frac{3}{8}$?

5. If a team ploughs $\frac{5}{7}$ of an acre in 6 hours, how much will it plough in one hour? How much in 3 hours?

What is $\frac{1}{6}$ of $\frac{6}{7}$? What is $\frac{1}{2}$ of $\frac{6}{7}$?

6. If a horse runs $\frac{1}{3}$ of a mile in one minute, how far will he run in $\frac{3}{4}$ of a minute?

How far will he run in $\frac{6}{7}$ of a minute?

What is $\frac{3}{4}$ of $\frac{1}{3}$? What is $\frac{6}{7}$ of $\frac{1}{3}$?

7. A man has $\frac{7}{8}$ of a dollar, which he wishes to distribute equally among several persons, giving $\frac{3}{16}$ of a dollar to each. How many can receive this sum? and what will be the remainder?

How many times is $\frac{1}{16}$ contained in 7? $\frac{3}{16}$ in 7? $\frac{3}{16}$ in $\frac{7}{8}$?
How many times is $\frac{1}{15}$ contained in 4? $\frac{2}{15}$ in 4? $\frac{2}{15}$ in $\frac{4}{7}$?
How many times is $\frac{1}{20}$ contained in 6? $\frac{7}{20}$ in 6? $\frac{7}{20}$ in $\frac{6}{7}$?

8. A man gave $\frac{6}{7}$ of a bushel of oats to some horses, giving to each $\frac{1}{8}$ of a bushel. To how many did he give it? and what was the remainder?

How many times will $\frac{1}{16}$ go in 5? In $\frac{4}{7}$? How many times will $\frac{2}{16}$ go in $\frac{6}{7}$?

9. A man has $\frac{7}{8}$ of a dollar. He gives $\frac{1}{4}$ of a dollar to one person, and $\frac{3}{8}$ of a dollar to a second. What part of a dollar has he left?

How many cents had he at first? How many cents did he give away? How many cents had he left?

10. If 13 pounds of figs cost $\frac{3}{8}$ of a dollar, what is that a pound?

11. If $5\frac{1}{2}$ lbs. of figs cost $\frac{5}{12}$ of a dollar, what is that a pound? Find first what one half pound will cost.

12. If $\frac{6}{7}$ of a cwt. of iron cost $4\frac{1}{3}$ dollars, what will a hundred weight cost?

13. If $34\frac{1}{2}$ lbs. of tea cost $11\frac{3}{8}$ dollars, what will 1 pound cost?

Here you find $\frac{69}{2}$ pounds cost $\frac{91}{8}$ of a dollar. Therefore 69 pounds must cost $\frac{91}{4}$ of a dollar.

14. If $\frac{2}{3}$ of a barrel of flour cost $3\frac{3}{8}$ dollars, what is that a barrel?

15. If wood is $5\frac{1}{2}$ dollars a cord, what will $\frac{7}{16}$ of a cord cost? What will $4\frac{1}{2}$ cords cost?

16. If $33\frac{1}{2}$ gals. of molasses cost $11\frac{3}{8}$ dollars, what is that a gallon?

17. If $31\frac{1}{2}$ gals. of vinegar cost $4\frac{5}{8}$ dollars, what is that a gallon?

18. If a bottle of wine, containing $1\frac{1}{2}$ pints, cost $\frac{5}{8}$ of a dollar, what would a barrel of wine come to at that rate?

19. In a pile of wood there are $13\frac{1}{2}$ cords. How many loads, of $\frac{3}{4}$ of a cord each, are there in the pile?

20. How many times will $2\frac{1}{4}$ go in $7\frac{1}{2}$? In $9\frac{1}{4}$? In 11?

21. How many loaves, of $8\frac{1}{2}$ oz. of flour each, can be made from 7 pounds of flour?

22. If a family consume $3\frac{1}{2}$ pounds of flour a day, how long will a barrel of flour, that is, 196 pounds, last them?

How long will it last if they consume $2\frac{1}{4}$ lbs. a day?

23. If a barrel of flour last a family 40 days, how long will 14 pounds last them?

24. A garrison of 100 men is allowed 12 oz. of flour a day to each man. How long will 10 barrels last them?

25. Two men hire a horse, a week, for 5 dollars. One travels with him 30 miles, the other 45 miles. What ought each to pay?

26. Two men hire a pasture in common for $4.80. One pastures his horse in it $7\frac{1}{2}$ weeks; the other pastures his horse 9 weeks. What ought each to pay?

27. A boy bought 3 doz. of oranges for $37\frac{1}{2}$ cents, and sold them for $1\frac{1}{2}$ cents apiece. What did he gain?

28. A man bought 7 yds. of cloth for 16 dollars, and sold it for 3 dollars a yard? What did he gain on each yard?

29. A man, worth 1690 dollars, left $\frac{2}{5}$ of his property to his wife. How much did she receive? The remainder he divided equally among 3 sons. What did each one receive?

30. A man bequeathed his estate of 14,000 dollars, one third to his wife, and the remainder to be divided equally among four sons. What did the wife, and what did each son, receive?

31. In an orchard one third of the trees bear apples, two fifths as many bear plums, and the rest bear cherries. What portion of the trees bear plums? What portion bear cherries? The number of cherry-trees is 40. What is the whole number of trees in the orchard?

32. What is $\frac{3}{7}$ of 549? What is $\frac{8}{9}$ of 374?

33. What is $\frac{1}{3}$ of $175\frac{1}{2}$? What is $\frac{5}{6}$ of 198?

34. What is $\frac{1}{2}$ of $\frac{3}{5}$ of 1640? What is $\frac{2}{3}$ of 972?

35. If 2 barrels of flour cost $11\frac{1}{2}$ dollars, what will 17 barrels cost? What will $22\frac{1}{2}$ barrels cost?

36. If $2\frac{1}{2}$ cords of wood cost 15 dollars, what will $68\frac{3}{4}$ cords cost? What will 200 cords?

37. If a horse eat $2\frac{1}{4}$ tons of hay in 30 weeks, what part of a ton will he eat in 1 week?

38. What is the cost of $23\frac{1}{2}$ yds. of cloth at $\frac{7}{8}$ of a dollar a yard?

39. What is the cost of $31\frac{1}{2}$ gallons of molasses, at $\frac{5}{16}$ of a dollar a gallon?

40. A grocer drew from a cask, containing $31\frac{1}{2}$ gallons, $\frac{1}{4}$ of its contents. Now, how much did he draw out? How much remained?

SECTION X.

THE LEAST COMMON MULTIPLE.

Name the multiples of 2 up to 20.
Name the multiples of 5 up to 30.
Name the multiples of 7 up to 63.
Name the multiples of 3 up to 15; also of 4 up to 20.

What one of the numbers just named is a multiple both of 3 and of 4?

Any product which has two or more numbers as factors of it is a *common multiple* of those numbers.

Name the multiples of 6 up to 24, and of 9 up to 27, and state what common multiple you find of 6 and 9.

Name the multiples of 8 up to 32, and of 12 to 36, and state what common multiple you find of 8 and 12.

Name all the multiples of 6 and of 8 below the number 50. How many common multiples have you found of 6 and 8?

What is the least common multiple of 6 and 8?

Find a common multiple of 6 and 9.

Find a second common multiple of 6 and 9.

What is the least common multiple of 6 and 9?

What is the least common multiple of 8 and 10?

What is the least common multiple of 3, 4, and 6?

What is the least common multiple of 4, 6, and 8?

From the above exercise it will be seen that the least common multiple of two or more numbers is the smallest product that contains them all as factors.

The *least* common multiple will always be the first one found in enumerating the series, as in the above examples.

But this method is often tedious; and we now proceed to exhibit another method, independent of any such series.

Suppose, now, we wish to find the smallest common multiple of 3 and 5. The number, it is clear, must be a certain number of 3's, and also a certain number of 5's. Now, by multiplying 3 and 5 together, we evidently obtain such a number; for it will be 3 times 5, and it will be 5 times 3. Multiplying the two numbers together, then, will always give their common multiple. The next question is, Will this product of

the two numbers be their least common multiple? This will depend on the character of the numbers. If the numbers are prime to each other, their product will be their least common multiple. For example, in the numbers 3 and 5, if we take any number of 5's less than 3, as 2×5, the factor 3 has disappeared, and the number is no longer a multiple of 3. If we take any number of 3's less than 5, as 4×3, the factor 5 has disappeared, and the number is no longer a multiple of 5. The product, therefore, of numbers prime to each other, is their least common multiple. In the above example, the numbers 3 and 5 were prime in themselves, and not merely prime to each other. To make the principle more clear, we will take two numbers that are not prime in themselves, but are only prime to each other.

What is the least common multiple of 8 and 9? Multiplying them together, we have 72. 72 is, then, a common multiple of 8 and 9. The question is, Is it their smallest common multiple? Writing the numbers with their factors, they are 2×2×2, and 3×3. Now, if we erase one of the 2's, we have no longer the factors of 8, and the product of the factors will not be divisible by 8. In the same way, if we erase one of the 3's, the product will not be divisible by 9.

If, then, the numbers are either prime, or prime to each other, the product is their least common multiple.

Next, let us inquire, What is the least common multiple of 4 and 6? Their product is 24; but this is evidently not their least common multiple, for 12 contains both 4 and 6 as factors. To show why it is, that, in this case, something less than the product of the numbers is their least common multiple, we will express each by its factors, thus, 2×2, 2×3. Now, it is clear that any number of times which you take 2×2 as a factor will be a multiple of 2×2. If, then, we throw out the 2 in the 2×3, and multiply by the

remaining 3, the product will be a multiple of 2×2, or 4. Looking, now, at the 2×3, or 6, it is evident that any number of times which you may take that as a factor will be a multiple of 2×3. But the 2 we may take from the 2×2, throwing away that in the 2×3; this leaves us to multiply the 2×3 by 2; as we before multiplied the 2×2 by 3, making 12 as the least common multiple. The rule, therefore, is: Retain of each prime factor the highest power which appears in any of the given numbers; erase the rest, and multiply together what then remain.

Find the least common multiple of 8, 24, and 36. Expressed by the factors, they are $2\times2\times2$; $2\times2\times2\times3$; $2\times2\times3\times3$. Now, $2\times2\times2$ is common to 8 and 24; it may be thrown out of the latter, leaving only 3. Examining again, you observe that 2×2 is common to 8 and 36; we throw this out of 36, leaving 3×3. Finally, 3, we find, is common to 24 and 36; throwing this out of 24, we find the numbers appear as follows: $2\times2\times2$; $\cancel{2}\times\cancel{2}\times\cancel{2}\times\cancel{3}$; $\cancel{2}\times\cancel{2}\times3\times3$. These multiplied together give for the least common multiple, 72. This conforms to the rule; for $2\times2\times2$ is the highest power of the factor 2, and 3×3 of the factor 3. What is the least common multiple of 24, 60, and 100? These factors are $2\times2\times2\times3$; $2\times2\times3\times5$; $2\times2\times5\times5$. We see that 2×2 is common to them all; expunge it in the second and third number. Next, 3 is common to the 1st and 2d; expunge it in the 2d. Lastly, 5 is common to the 2d and 3d; expunge it in the 2d, and the numbers will stand, $2\times2\times2\times3$; $\cancel{2}\times\cancel{2}\times\cancel{3}\times\cancel{5}$; $\cancel{2}\times\cancel{2}\times5\times5$. These multiplied together give 600.

To multiply these most easily, first take $2\times2\times5\times5=100$; then the remaining factors, 2×3, multiplied by 100, give 600.

What is the least common multiple of 24, 40, and 72?

What is the least common multiple of 18, 54, 81?
What is the least common multiple of 15, 4, 7? Of 15, 40, 27? Of 16, 14, 6? Of 60, 12, 18?

From the foregoing reasoning and examples, you will perceive that the least common multiple of several numbers is the product of all their prime factors, each taken in the highest power in which it appears in any of the numbers.

SECTION XI.

PRACTICAL QUESTIONS.

1. What part of a shilling is 1 penny? 2 pence? 3 pence? 4 pence? 5 pence? 6 pence? 7 pence?
2. What part of a penny are 2 farthings? 3 farthings? 4 farthings? 5 farthings? 6 farthings? 8 farthings?
3. What part of a shilling is 1 farthing? 2 farthings? 3 farthings?
What part of a shilling is 1 penny and 1 farthing? 1 penny 2 farthings? 3d. 3qrs.? 4d. 2qrs.? 6d. 1 qr.? 9d. 2qrs.?
4. What part of a pound is 1 shilling? 2s.? 3s.? 5s.? 1s. 1d.? 2s. 1d.? 4s. 3d.? 5s. 6d.? 7s. 9d.? 3s. 8d.?
5. What part of a pound is 1 farthing? 2 qrs.? 3 qrs.? 2d. 3 qrs.? 5d. 2 qrs.? 1s. 1d. 1 qr.? 6s. 7d. 3 qrs.?
6. What part of a pound avoirdupois is 2 oz.? 3 oz.? 4 oz.? 5 oz.? 6 oz.? 7 oz.? 8 oz.? 9 oz.? 10 oz.?
7. What part of one ounce is one dram? What part

of one pound is one dram? 2 drs.? 3 drs.? 1 oz. 1 dr.? 1 oz. 2 drs.? 2 oz. 4 drs.? 3 oz. 6 drs.? 8 oz. 3 drs.? 9 oz. 11 drs.?

8. What part of a pound is $\frac{1}{16}$ of an oz.? $\frac{3}{16}$ of an oz.?

What part of a pound is $\frac{1}{2}$ an oz.? $2\frac{1}{2}$ oz.? $3\frac{1}{2}$ oz.? $4\frac{1}{2}$ oz.?

9. What part of a pound Troy is 1 dwt.? 5 dwt.? 6 dwt.? 9 dwt.? 11 dwt.? 10 dwt.? 1 oz. 1 dwt.? 3 oz. 4 dwt.?

What part of an oz. Troy is 1 dwt.? 3 dwt. 1 gr.? 4 dwt. 6 gr.? 7 dwt. 3 gr.? 8 dwt. 9 grs.? 10 dwt.? 12 dwt.? 16 dwt.?

10. What part of an ell English is 1 qr. of a yard? 2 qrs.? 3 qrs.? What part of a qr. is 1 nail? 3 nails?

11. What part of a yd. is 1 qr. 1 nail? 2 qrs. 3 n.? 3 qrs. 2 n.? What part of an ell English is 3 nails? 1 qr. 3 n.? 4 qrs. 1 n.?

12. What part of a yd. is 1 inch? 4 inches? 7 inches? 9 inches? What part of a yard is 1 qr. 2 in.? 2 qrs. 3 in.? 3 qrs. 1 in.?

13. From a vessel containing 3 gallons of wine 3 gills leaked out. What part of a gallon leaked out? What part of a gallon remained?

14. From a barrel full of wine 7 quarts were drawn. How many quarts remained? What part of the barrel had been drawn out? What part of the barrel had remained?

15. If $\frac{2}{3}$ of a barrel of beer be divided into 4 equal parts, what part of a barrel will each of the parts be? How many gallons will each part be?

16. If one quart be taken from a barrel full of beer, what part of a barrel will remain? If 3 pints be taken out, what part will remain? If $7\frac{1}{2}$ gallons be taken out, what part of a barrel is taken out? What part of a barrel remains?

17. A man distributed $7\frac{1}{2}$ gallons of milk among 5 persons. What part of a gallon did he give to each?

18. If you have $3\frac{1}{2}$ gallons of milk, and distribute it to some poor persons, giving $\frac{2}{5}$ of a gallon to each, how many persons will you give it to? How much will remain?

19. What part of 1 foot is $1\frac{1}{2}$ in.? $2\frac{1}{2}$ in.? $5\frac{1}{2}$ in.? $6\frac{1}{4}$ in.? $8\frac{3}{4}$ in.? $9\frac{1}{4}$ in.? $10\frac{2}{3}$ in.? $11\frac{1}{4}$ in.?

20. What part of a yard is 2 inches? $3\frac{1}{2}$ inches? 14 in.? $5\frac{1}{4}$ in.? $6\frac{1}{4}$ in.? $17\frac{1}{2}$ in.? $24\frac{1}{4}$ in.?

21. What part of a rod is $\frac{1}{2}$ a foot? $1\frac{1}{2}$ feet? $2\frac{1}{2}$ feet? 4 ft. 3 in.? 6 ft. 7 in.? 10 ft. 5 in.?

22. What part of 3 rods is $\frac{1}{2}$ a foot? 1 foot? $3\frac{1}{2}$ feet? What part of a furlong are $2\frac{1}{2}$ rods? $5\frac{1}{4}$ rods?

23. What fraction of a foot is $\frac{1}{7}$ of a yard? $\frac{2}{7}$ of a yd.? What fraction of a foot is $\frac{1}{13}$ of a rod? $\frac{2}{13}$ of a rod? $\frac{3}{13}$ of a rod?

24. A man measured the length of his barn with a stick half a yard long, and found the barn $31\frac{1}{2}$ times the length of his stick. How long was it?

25. A carpenter is cutting up a board, $17\frac{1}{2}$ feet in length, into pieces $2\frac{1}{4}$ feet long. How many pieces will there be, and how long will be the piece that remains?

26. A man measures a piece of fence with a pole $9\frac{1}{2}$ feet long. The fence is $15\frac{1}{2}$ times the length of the pole. How many rods is it in length?

27. What part of a peck is $\frac{1}{50}$ of a bushel?

What part of a gallon are $\frac{4}{17}$ of a peck? $\frac{3}{4}$ of a peck?

What part of a quart is $\frac{1}{22}$ of a peck? $\frac{2}{23}$ of a peck?

What part of a quart are $\frac{4}{75}$ of a bushel? $\frac{2}{28}$ of a bushel?

28. What part of a peck is $\frac{1}{8}$ of a bush.? $\frac{2}{8}$ of a bush.? $\frac{3}{8}$ of a bush.? $\frac{4}{8}$ of a bush.? $\frac{5}{8}$ of a bush.?

29. Two men bought a lot of standing wood in company, for 11 dollars. One cut off 2 cords, the other 1 cord. What ought each to pay?

30. Two boys bought the chestnuts on a tree for 50 cents. One had 11 quarts, the other 6 quarts and 1 pint. What ought each to pay?

31. Three men bought a piece of cloth for 24 dollars. The first took $2\frac{1}{2}$ yds., the second the same quantity; and on measuring the remainder, it was found to be 3 yards. What ought each to pay?

32. Two men hire a horse for a month for 12 dollars. One travels 200 miles with the horse, the other 150. How much should each pay?

SECTION XII.

DECIMAL FRACTIONS.

[See Numeration, Part II.]

In the calculations in common fractions, a great inconvenience arises from their irregularity. There is no law regulating the magnitude of either of the terms. The denominator may be in any ratio whatever to the numerator. From seeing one you can make no inference at all respecting the magnitude of the other. In calculations of addition, it is often more than half the work to bring the fractions into a common denomination.

Now, it is evident, that, if fractions could be written in the same manner as whole numbers, that is, increasing in a tenfold rate as you advance to the left, and decreasing in a tenfold rate as you advance to the right, an immense gain would be made in the convenience of calculating them. Operations in fractions would then be just as easy as operations in whole numbers. Now, this advantage is gained in decimal fractions. They are brought under the same law as

whole numbers. Let us observe the manner in which whole numbers are written. Take the number 222; the right-hand figure signifies two units, the next two tens, the next two hundreds; just as if it were written in this manner, $2\times100+2\times10+2$; two multiplied by 100, plus two multiplied by ten, plus two; making two hundred and twenty-two. But this cumbersome method of writing is unnecessary, because the law of notation determines what number the figures in each place shall be multiplied by. It must not be forgotten that the figure 2, in the above example, in no case signifies of itself more than two. It is the place it occupies that gives it the higher value of tens or hundreds.

Now, it would evidently be a great convenience if we could reduce fractions to the same law, so that they would, like whole numbers, decrease in a decimal ratio, in advancing from the left to the right. To show this regularity to the eye, we will write the following numbers: two multiplied by 1000, two multiplied by 100, two multiplied by 10, two units, two divided by 10, two divided by 100, and two divided by 1000. Written in full, they would stand thus: $2\times1000+2\times100+2\times10+2+\frac{2}{10}+\frac{2}{100}+\frac{2}{1000}$.

But we have seen that we may write the whole numbers without the multipliers, thus, 2222, because we know, from the place each figure occupies, what its multiplier must be. Just so we can write fractions without the denominators, provided we know, from the place of the numerator, what the denominator must be. Thus the whole of the above series may be written as follows; 2222.222. A *decimal*, therefore, is the numerator of a fraction, whose denominator is never written, but is always understood to be 1 with as many ciphers as there are places in the decimal.

You observe that, in writing the series given above,

there is a period placed at the right hand of the whole numbers, separating the unit figure from that of tenths. The period must never be omitted when there are fractions, for it enables you to determine the value of each figure in the sum. Instead of reading .22 two tenths and 2 hundredths, we may call it 22 hundredths, which is more convenient, and amounts to the same; for two tenths is equal to 20 hundredths; so .222 is two hundred and twenty-two thousandths. So, in all cases, read the *decimal* numbers as whole numbers, and for their denominator take 1 with as many ciphers as there are places in the written decimals.

In all your study of decimals, be careful not to confound the words which express fractions with the similar words which express whole numbers; as *tenths* with *tens*, *hundredths* with *hundreds*. The following questions will aid you in fixing this distinction clearly in mind:

1. How many tenths are equal to ten whole ones?
2. How many tenths are equal to two and a half whole ones?
3. How many hundredths are equal to three and a quarter whole ones?
4. How many hundredths are equal to one hundred whole ones?
5. How many thousandths are equal to ten whole ones?
6. In fifteen whole ones how many tenths? How many hundredths?
7. In seventy-five hundredths how many tenths?
8. In three tenths how many hundredths?
9. In six tenths how many thousandths?

Thus, you observe, fractions have been brought under the same law that regulates the writing of whole numbers. They may now be added, sub-

tracted, multiplied, and divided, like whole numbers. But, in doing this, it is important to determine the place of the period that separates the whole numbers from the fractional part of the sum. Where must the period be placed in the answer?

ADDITION AND SUBTRACTION OF DECIMALS.

Let us first observe how important it is that the rule in this case be entirely correct. If I have this number, 32.5, to write, and by any mistake I should write it 3.25, it would denote a quantity only one tenth as great as it should be; or, if I should write 325. it would denote a quantity ten times greater than it should be. Moving the period one place to the right, makes the number ten times as grêat as it was before; for tens become hundreds, and hundreds, thousands; and each figure ten times as great as before. So, by moving the period one place to the left, the number becomes just one tenth what it was before. Removing the period two places from its true place, makes the number 100 times larger or smaller than it should be, according as you remove it to the right or the left. Hence you may see that, in order to multiply a number that has decimals, by 10, you have only to remove the period one place to the right; to multiply by 100, remove it two places; and so on. To divide by 10, remove the period one place to the left; to divide by 100, remove it two places; and so on. From the above you may see the importance of being perfectly accurate in fixing the place of the decimal in the answer to any question.

We will begin with addition. Add 4.46 to 3.21. Here you observe the two whole numbers make 7, and 46 hundredths added to 21 hundredths make 67 hundredths. The answer, then, must be 7.67, having

two decimal places. Add 6.8 to 5.23. The 3 hundredths must evidently stand alone, since there is nothing like it to add to it; 2 tenths added to 8 tenths make 10 tenths, or one whole one; this we carry to the 5, which gives us for the answer, 12.03. This will serve to suggest the rule for placing the period in the answer to questions in addition. The number of decimal places in the answer must be as great as can be found in any one of the numbers to be added.

The same rule holds in subtraction. Take for illustration the numbers given in the second example of addition. From 6.8 subtract 5.23. Now, as in the minuend there are no hundredths, we must borrow 10 in this place, and we shall have a remainder of 7 hundredths; adding 1 tenth to the subtrahend, to compensate for the 10 hundredths added to the minuend, we have, in the place of tenths, a remainder of 5; finally, in the place of units we subtract 5 from 6; the answer is 1.57. In performing this operation, you may, if you please, call the 8 tenths 80 hundredths; then 23 hundredths from 80 hundredths leaves 57 hundredths. By performing slowly and with care examples of your own selection, you will see the verification of the rule given above, both for addition and subtraction.

Add 2.4 to 3.8. Add .6 to 1.3. Add .4 to .3. Add .37 to .25. Add 3.7 to .24. Add 1.08 to .05.

From 4.6 subtract 2.4. From 7.1 subtract 6.4. From .18 subtract .13. From 4.5 subtract .6.

In these examples, each step should be explained by the pupil as he performs it.

MULTIPLICATION OF DECIMALS.

The rule in multiplication we shall find to be different from the above.

1. First, we will multiply 2.4 by 3. If we regard the multiplicand as a whole number, the answer will be 72. But by regarding the multiplicand as a whole number, — as 24 instead of 2 and 4 tenths, — we regarded it as ten times greater than it really is. The answer, therefore, is ten times too great. Instead of 72, it must be 7.2.

2. Multiply 6.2 by 3.4. By regarding both as whole numbers, we obtain the answer 2108. Now, in calling the multiplicand 62 instead of 6.2, we treated it as 10 times greater than it is. The answer must therefore be 10 times too great, even if the multiplier were a whole number. We must therefore divide it by 10, or write 210.8. But the multiplier also is 10 times too great; the answer must therefore be divided again by 10, in order to bring it right. Thus the answer will stand 21.08.

3. Again; multiply .62 by 3.4. Here we obtain the same figures as before, 2108; but, by treating the multiplicand as a whole number, we regarded it as 100 times too great; the answer, therefore, must be divided by 100, or written 21.08. But the multiplier, calling it a whole number, was taken 10 times greater than it is; the answer must be again divided by 10, and thus it will stand 2.108.

4. Once more; multiply .62 by .34. The figures of the answer are, as before, 2108; but, by regarding both the factors as whole numbers, we take each 100 times greater than it is. We must therefore divide by 100 to correct the error in the multiplier, and again by 100 to correct the error in the multiplicand. This will remove the point four places to the left, and the true answer will be .2108. By examining these examples, you will see that the pointing in each case conforms to the following rule:

Point off as many figures for decimals in the answer

as there are decimal places in both the factors taken together.

5. Multiply 2.7 by .3.—6. Multiply .6 by .7.—7. Multiply 6. by .7.—8. Multiply .02 by .3.—9. Multiply .02 by .03.—10. Multiply .01 by .01.

DIVISION OF DECIMALS.

1. Divide 48 by 12. *Ans.* 4.

2. Divide 4.8 by 12. The figure expressing the answer is 4, as in the first case; but, observe, the dividend is only one tenth as large as before; the quotient, therefore, is only one tenth as large. Instead of 4. it is .4.

3. Divide .48 by 12. The figure of the quotient is still 4; but, as the dividend is only one hundredth part as large as in the first example, the quotient will be only one hundredth part of 4, or 4 hundredths, written thus, .04.

4. Again; divide 48 by 1.2. The quotient is still 4; but we must investigate the question to see where this 4 must stand. You observe that the divisor is now only one tenth of 12. Now, if the divisor is only one tenth as great as it was before, you must consider how that will affect the quotient. You will perceive, on reflection, that, as you diminish the divisor, you increase the quotient. If you make the divisor half as great, the quotient will be twice as great; and so proportionally of other numbers. Now, as, in this instance, the divisor is one tenth as great as before, the quotient must be ten times greater. The figure 4, then, which is the quotient figure, instead of standing in the place of units, as before, must stand in the place of tens; that is, it must be 40, the cipher merely showing that the 4 stands in the place of tens.

5. Once more; divide 48 by .12. Here, again, you have 4 for the quotient figure, for you can have no other; but, on comparing this example with the first, you perceive the divisor is only one hundredth part as great; the quotient must therefore be one hundred times greater; that is, it is 400, the ciphers merely removing the 4 into the place of hundreds.

On examining these examples carefully, you will see that each answer is unquestionably correct. "But by what rule," you ask, "are these examples wrought?" They are not wrought by rule, but by reasoning on the numbers themselves; and the more you habituate yourself to reason in arithmetic, the less need you will have to depend on rules.

With this suggestion I will now state a rule, which you may at any time follow, when you have not time to look into the reason of the operation.

There must be as many decimals in the quotient as the decimals in the dividend exceed those in the divisor. When there are fewer decimals in the dividend than there are in the divisor, ciphers must be added so as to make the number equal.

We will now review the foregoing examples, and observe their conformity with the above rule. Example 1 has no decimals in the divisor or the dividend, therefore none in the quotient. Ex. 2, the dividend has one decimal, the divisor none; the quotient has therefore one. Ex. 3, the dividend has two decimals, the divisor none; the quotient has two. Ex. 4, the dividend has none, the divisor one; there must then be a cipher added to the dividend, and then the quotient will be in whole numbers. Ex. 5, the dividend has none, the divisor two; there must then be two ciphers added, and then the quotient will be in whole numbers.

6. Divide 45 by 15. Divide 4.5 by 15. Divide .45 by 15. Divide 45 by 1.5. Divide 45 by .15.

7. Divide 66 by 11; 6.6 by 11; .66 by 11; 66 by 1.1; 66 by .11.

In calculations of Federal money, cents and mills are regarded as decimals; the point, therefore, separating the whole numbers from the fractions must be placed between the dollars and the cents. Thus, 24.00 is 24 dols.; 2.40 is 2 dols. 40 cts.; 0.24 is 24 cts.

8. A man divided $24.00 among 3 men. How much did each receive?

9. A man divided $2.40 among 3 men. How much did each receive? Divide 2.4 by 3.

10. A man divided $0.24 among 3 men. How much did each receive? Divide $0.24 by 3.

11. A man divided 36 dollars among 4 persons. How much did each receive? Divide 36 by 4.

12. A man divided $3.60 among 4 persons. How much did each receive? What is one fourth of $3.60?

13. A man divided $0.36 among 4 men. How much did each receive? What is one fourth of .36?

SECTION XIII.

REDUCTION OF VULGAR FRACTIONS TO DECIMALS.

We have now seen that decimal fractions have this great advantage over vulgar fractions,—that they conform to the same law of notation as whole numbers, and may be added, subtracted, multiplied, and divided, in the same manner, and with the same ease, as whole numbers. It is desirable, therefore, to introduce them in a great many cases instead of vulgar fractions. The next question that arises, therefore, is, Can a vulgar fraction be changed to a decimal having the same value; and how can it be done? Take the fraction

$\frac{1}{2}$; we wish to reduce it to tenths, or, in other words, to express it in tenths. Now, we can change any number to tenths by multiplying it by 10. Thus, 3 is 30 tenths, 4 is 40 tenths. We will now take $\frac{1}{2}$, and change the numerator, 1, to tenths, and it will stand 1.0; but the fraction was not one, but one half of one; 1.0, therefore, is twice as great as it should be; we must divide it, therefore, by 2, that is, by the denominator, and it will be .5. To reduce a vulgar fraction, then, to a decimal; add a cipher to the numerator, and divide by the denominator. If one cipher is not enough to render the division complete, add more.

Reduce to a decimal $\frac{1}{5}$. Change the numerator to tenths; it will be 1.0; but the quantity to be reduced to tenths was not one, but one fifth of one; 1.0, therefore, is 5 times greater than it should be; dividing by 5, the answer is .2.

Reduce to a decimal the fraction $\frac{2}{5}$, explaining each step in the operation.

Reduce to a decimal the fraction $\frac{4}{5}$.

Reduce to a decimal the fraction $\frac{3}{5}$.

Reduce to a decimal the fraction $\frac{1}{4}$.

Reduce to a decimal the fraction $\frac{3}{4}$.

I will here direct your attention to a fact that it is interesting to notice. If the denominator of the vulgar fraction is one of the factors of 10, that is, if it is either 2 or 5, the decimal figure will be as many times the other factor as there are units in the numerator of the vulgar fraction. This will appear self-evident when we express the numbers by their factors. Thus, in obtaining the decimal for $\frac{1}{2}$, we divide 10 by 2; but 10 is 2×5; therefore, in dividing by 2, we simply expunge the factor we divide by, and leave the other; $2)\cancel{2}\times5$. So in the fraction $\frac{1}{5}$, we obtain the decimal by dividing 10 by 5, which expunges the factor 5; $5)\cancel{5}\times2$. In reducing $\frac{2}{5}$, we divide 2×10 by 5, thus, $5)2\times2\times\cancel{5}$, leaving twice the factor 2; in $\frac{3}{5}$, $5)3\times2$

$\times\cancel{5}$, leaving 3 times the factor 2; in $\frac{4}{5}$, $5)2\times2\times2\times\cancel{5}$, leaving 4 times the factor 2.

2. We will now take the fraction $\frac{1}{4}$. Proceeding as before, we wish to divide 10 by 4, thus, $2\times2)2\times5$. Here we see the division cannot be complete; for the divisor contains the factor 2 twice, while the dividend has it only once. If, however, we had multiplied the original numerator, 1, by 100, instead of 10, we should have had 10 twice as a factor in the dividend, and of course each factor of 10 twice; 100 is 10×10, and 10 is 2×5. It would have stood then thus, $2\times2)2\times5\times2\times5$. The division is now complete; for the dividend contains the factor 2 as many times as the divisor has it. Expunging these, we have remaining the factor 5 taken twice, or .25.

This process, you may observe, conforms to the rule, to add as many ciphers as may be necessary to render the division complete.

3. Reduce the vulgar fraction $\frac{3}{4}$ to a decimal. 30 is composed of the prime factors $3\times2\times5$; it contains 2 only once, and therefore it is not divisible by 2×2; 30 must therefore be multiplied by 10. This will introduce another 2, and it will stand thus, $2\times2)3\times2\times5\times2\times5$. By expunging the two 2's, and multiplying together the other factors, we have .75 for the answer.

4. Reduce the fraction $\frac{1}{8}$ to a decimal. 10, expressed by its factors, is 2×5, and 8 is $2\times2\times2$. We must therefore multiply 2×5 by 10 till it shall contain the factor 2 as many times as 8 contains the same factor; that is, the numerator, 1, must be multiplied by a thousand. It will then stand, $2\times2\times2)2\times5\times2\times5\times2\times5$. Expunging the three 2's, there remains for the answer, .125.

By examining the above examples, you may observe this fact,—that if the denominator of the vulgar fraction contains one of the factors of 10, that is, 2 or 5, one

or more times as a factor, the decimal will contain the other factor just as many times. Thus, $\frac{1}{2}=.5$; $\frac{1}{4}$ or $\frac{1}{2\times2}=.25$, or $.5\times.5$; $\frac{1}{8}$ or $\frac{1}{2\times2\times2}=.125$, or $.5\times.5\times.5$. In the same way, $\frac{1}{5}=.2$; $\frac{1}{25}$ or $\frac{1}{5\times5}=.04$, or $.2\times.2$; $\frac{1}{125}$ or $\frac{1}{5\times5\times5}=.008$, or $.2\times.2\times.2$. In this way you may determine that $\frac{1}{16}$, when reduced to a decimal, will contain 5 four times as a factor, because 16 contains 2 four times as a factor. So $\frac{1}{32}$ will contain 5 five times as a factor.

This is conveniently expressed by saying, whatever power of one of the factors of 10 the denominator of the vulgar fraction contains, the same power of the other factor will appear in the decimal.

5. Reduce $\frac{1}{3}$ to a decimal fraction. Preparing the numbers as before, it will stand $3)2\times5$. You observe that 3 is different from either of the factors of 10. Now, as 10 has only the factors 2 and 5, it is not divisible by 3 without a remainder.

If you add to the numerator ever so many ciphers, you will only increase the number of times that 2 and 5 appear in it as its factors, and the number can never become divisible by 3 without a remainder. The answer becomes .333+, and this indefinitely, as far as you may please to carry on the operation. On the same principle, we shall find that it is not possible to express accurately, in decimals, any vulgar fraction whose denominator contains as a factor any thing different from the factors of 10; for this denominator becomes, in the reduction, a divisor of 10 or some power of 10; and if it has any thing in it as a factor which is prime to the factors of 10, the complete division is impossible. Thus $\frac{1}{6}$ cannot be exactly expressed in decimals; because, though one of its factors, 2, is a divisor of 10, the other, 3, is prime to 10. On this principle the following questions may be examined.

Can $\frac{1}{7}$ be accurately expressed in decimals? Why?
Can $\frac{1}{8}$ be accurately expressed in decimals? Why?

Can $\frac{1}{9}$ be accurately expressed in decimals? Why?

Can $\frac{1}{12}$ be accurately expressed in decimals? Why?

Can $\frac{1}{20}$? $\frac{1}{15}$? $\frac{1}{24}$? $\frac{1}{25}$? $\frac{1}{30}$? $\frac{1}{50}$? $\frac{1}{80}$? $\frac{1}{92}$? $\frac{1}{75}$? $\frac{1}{13}$? $\frac{1}{14}$? $\frac{1}{16}$? $\frac{1}{35}$? $\frac{1}{17}$?

6. Name all the denominators, from 2 up to 20, of such fractions as can be accurately expressed in decimals. From 20 to 40. From 40 to 60. From 60 to 80.

7. Name all the denominators, from 2 to 20, of such fractions as cannot be expressed accurately in decimals. From 20 to 40. From 40 to 60. From 60 to 80.

8. What is the value of 4 shillings, expressed in the decimal of a £? As 1 shilling is $\frac{1}{20}$ of a £, 4 s. is $\frac{4}{20}$. We can change 4 to tenths by adding a cipher; it will then be 40. 4, however, was not the number we wished to reduce to tenths, but $\frac{4}{20}$; the answer, 40, is therefore 20 times too great; dividing by 20, it stands .2. 4 shillings, then, is 2 tenths of a £.

9. Now, reverse the operation. What is the value, in shillings, of .2 of a £? Now, shillings are twentieths. We can change any number to twentieths by multiplying it by 20; as, 1 is 20 twentieths, 2 is 40 twentieths, &c. Multiplying the .2 by 20, we have 40; but observe the 2 was not two wholes, but two tenths; the answer, 40, therefore, is ten times too great. Dividing by 10, the answer is 4 shillings.

10. Reduce to the decimal of a £, 2 shillings. 5 shillings.

11. What is the value, in shillings, of .1 of a £? Of .25 of a £?

12. Reduce to the decimal of a shilling, 3 pence. 3 pence are $\frac{3}{12}$ of a shilling. Reducing to hundredths, to render the division complete, the answer is .25.

13. What is the value, in pence, of .25 of a shilling?

14. Reduce 9 pence to the decimal of a shilling.

15. Reduce 1 peck to the decimal of a bushel.

16. Reduce 3 pecks to the decimal of a bushel.

17. Reduce .5 of a bushel to pecks. .75 of a bu to pecks.

18. Reduce 15 minutes to the decimal of an hour.

19. Reduce 45 minutes to the decimal of an hour.

20. Reduce to minutes .5 of an hour. .25 of an hour. .75 of an hour.

21. Reduce 6 inches to the decimal of a foot. 9 inches to the decimal of a foot. 3 inches to the decimal of a foot.

SECTION XIV.

INTEREST.

Interest is the sum paid by the borrower to the lender for the use of money. The rate of interest is established by law, and varies in different countries. In England, it is 5 per cent., that is, 5 for the use of 100 for 1 year; in the New England States, it is 6 per cent.; in New York, it is 7 per cent. When no particular rate is mentioned in this book, 6 per cent. will be understood.

If I borrow 100 dollars for 1 year, at the end of the year I owe the sum I borrowed, 100 dollars, and 6 dollars for the use of it, making 106 dollars. The sum borrowed is the *principal;* the sum paid for the use of it is the *interest;* the principal and interest added together make the *amount.*

1. What is the interest of 100 dols. for 2 years? 3 years? 4 years? 5 years? 6 years? 7 years?

2. What is the interest of 200 dols. for 2 years? 3 years? 4 years? 5 years? 6 years?

3. What is the interest of 300 dols. for 2 years? For 4 years? Of 400 dols. for 3 years?

4. What is the interest of 50 dols. for 1 year? For 3 years? Of 25 dols. for 1 year? 2 years?

5. What is the interest of 100 dols. for 1 year?

What is the interest of 100 cents for 1 year?

What is the interest of 2 dols. for 1 year? Of 3 dols.? Of 4 dols.? 5 dols.? 6 dols.? 7 dols.? 8 dols.? 9 dols.?

6. What is the interest of 36 dols. for 1 year? Of 47 dols.? Of 57 dols.? Of 34 dols.? Of 62 dols.? Of 89 dols.? Of 125 dols.? Of 136 dols.? Of 207 dols.? Of 561 dols.?

7. What is the interest of 50 cents for 1 year? Of 25 cents? Of 10 cents? Of 20 cents? Of 30 cents? Of 40 cents? Of 50 cents? Of 70 cents? Of 80 cents? Of 90 cents?

8. What is the interest of 50 dols. 60 cents for 1 year? Of 84.30? Of 96.40? Of 112.25? Of 230.75?

9. What is the interest of 100 dols. for 6 months? For 3 months? For 2 months? For 1 month? For 4 months? For 5 months? For 7 months? For 8 months? For 9 months? For 10 months? For 11 months?

10. What is the interest of 10 dols. for 6 mo.? 3 mo.? 2 mo.? 1 mo.? 4 mo.? 5 mo.? 7 mo.? 8 mo.? 9 mo.? 10 mo.? 11 mo.?

11. What is the interest of 1 dol. for 6 mo.? 1 mo.?

The interest of 1 dollar for 1 month is half a cent, and for any number of months, it is half as many cents.

12. What is the interest of 1 dollar for 5 mo.? 7 mo.? 8 mo.? 9 mo.? 11 mo.? 12 mo.? 15 mo.? 16 mo.? 17 mo.? 18 mo.?

The interest of any number of dollars for 1 month is half as many cents.

13. What is the interest of 12 dollars for 1 mo.? Of 15 dols.? 25 dols.? 37 dols.? 42 dols.? 67 dols.? 93 dols.? 104 dols.?

14. What is the interest of 12 dols. for 3 months? What is the interest of 25 dols. for 6 months?

In computing interest, a month is reckoned 30 days. As the interest on a dollar for 30 days is half a cent, that is, 5 mills, the interest on a dollar for 1 fifth of 30 days will be 1 mill. One fifth of 30 is 6; the interest, therefore, on 1 dollar for 6 days is 1 mill; and the interest on any number of dollars for 6 days will be as many mills as there are dollars.

15. What is the interest of 15 dollars for 6 days? Of 25 dols.? Of 40 dols.? Of 65 dols.? Of 75 dols.? Of 100 dols.? Of 500 dols.? Of 360 dols.? Of 840 dols.? Of 1000 dols.?

As the interest of 1 dollar for 6 days is 1 mill, for 12 days it will be 2 mills, for 18 days 3 mills, &c.

16. What is the interest of 1 dol. for 24 days? Of 2 dols. for 6 days? Of 2 dols. for 12 days? Of 2 dols. for 18 days? Of 5 dols. for 6 days? For 12 days? For 24 days? Of 36 dols. for 18 days?

17. What is the interest of 125 dols. for 1 year and 6 mo.?

18. What is the interest of 268 dols. for 1 year? For 2 years? For 3 years?

19. What is the interest of 45 dols. for 4 years 7 mo.?

20. What is the interest of 60 dols. for 1 year 3 mo. 18 days?

21. What is the interest of 100 dols. for 2 years 1 mo. 12 days?

22. What is the interest of 165 dols. for 3 years 2 mo. 6 days?

23. What is the interest of 50 dols. for one month? For 6 months? For 1 year and 7 months?

24. What is the interest of 94 dols. for eight mo. 24 days?

25. What is the interest of 320 dols. for 8 mo. and 12 days?

26. What is the interest of 84 dols. for 4 mo. and 15 days?

27. What is the interest of 196 dols. for 10 mo.?

28. What is the interest of 86 dols. for 9 days?

29. What is the interest of 340 dols. for 15 days?

30. What is the interest of 875 dols. for 22 days?

When interest is more or less than 6 per cent., first find the interest at 6 per cent., and then make a proportional addition or subtraction for the required per cent. If it is 7 per cent., add one sixth; if 5 per cent., subtract one sixth.

31. What is the interest of 140 dols. for 1 year, at 7 per cent.?

32. What is the interest of 200 dols. for 1 year and 6 mo., at 5 per cent.?

33. What is the interest of 460 dols. for 1 year, at $4\frac{1}{2}$ per cent.?

Remark.—$4\frac{1}{2}$ is three fourths of 6.

34. What is the interest of 500 dols. for 1 mo., at 9 per cent.?

BANKING.

When money is obtained at a bank, the note which is given for it promises to pay it at a certain time, as 60, 90, or 120 days. The interest on this note, instead of being paid at the end of the time, when the note is taken up, is paid beforehand; that is, it is subtracted from the sum named in the note; so that,

when you take up the note, you have only to pay the face of it, as the interest has been paid already.

If you give a note to a bank for 100 dollars, to be paid in 90 days, they subtract from the sum named in the note the interest of the sum for 90 days, and three days besides, called *days of grace;* the balance is the sum you receive. The interest of 100 dollars for 90 days is $1.50; for 3 days, it is 5 cents. $1.55 subtracted from $100.00 leaves a balance of $98.45, which is the sum you will receive.

If the note is given for 60 days, the interest is cast for 63 days, and subtracted from the sum named.

The interest, thus subtracted, is called the *bank discount;* and the bank, when it lends money on such a note, is said to discount the note.

35. What is the bank discount on a note of 100 dollars, payable in 30 days? And how much will be received on such a note?

The interest on 100 dollars for 30 days is 50 cents; for 3 days, it is 5 cents; the discount, 55 cents, subtracted from 100 dollars, leaves $99.45, the sum received.

36. What is the bank discount on a note for 200 dollars for 60 days? And what is the cash value of the note?

37. What is the bank discount, and what is the cash value of a note for 150 dollars payable in 30 days?

38. What is the bank discount, and what is the cash value of a note for 200 dollars payable in 90 days?

39. What is the bank discount, and what is the cash value of a note for 300 dollars payable in 90 days?

DISCOUNT.

When money is paid by the debtor before it becomes due, an allowance is made, which is called *discount.* If I owe 100 dollars, to be paid in three months from this time, and I pay it now, I ought not to pay the full hundred dollars, for I am entitled to the use of the money three months longer. The sum which should be paid now, to cancel a debt due at some future time, is called the *present worth* of the debt.

To find the *present worth* of a debt due at some future time, first find the interest on the debt from the time of payment to the time when the debt is due; subtract this interest from the debt, and the remainder will be the *present worth.* Thus, if I pay a debt of 100 dollars three months before it is due, I subtract the interest of 100 dollars for three months (= $1.50) from 100 dollars, leaving $98.50 for the sum which I must pay.

This rule is not strictly equitable, because $98.50, with three months' interest added, will not amount to $100. The above method, therefore, gives the present worth a little too small; but it is the method uniformly adopted in business, and the error is on the right side, for it encourages the debtor to be prompt in his payments.

40. What is the present worth of 200 dollars payable in 1 year?

41. What is the present worth of 150 dollars payable in 2 years?

42. What is the present worth of 60 dollars payable in 6 months?

43. What is the present worth of 530 dollars payable in 1 year?

What is the present worth of 400 dollars payable in 1 year and 6 months?

LOSS AND GAIN.—PER CENTAGE.

44. A boy bought a penknife for 25 cents, and sold it for 28 cents. How many cents did he gain on a quarter of a dollar?

45. Suppose he had bought 4 knives at the same price each, and sold them at the same profit, he would then have traded with a dollar. How much would he have gained on a dollar?

This is called so much *per cent.*, which only means so much on a hundred.

46. A boy bought a bushel of apples for 50 cents, and sold them for 59 cents. How much did he gain per cent.?

47. A bookseller bought a book for 75 cents, and sold it for 84 cents. How much did he make per cent.?

As 75 is $\frac{3}{4}$ of 100, what he gained on the book will be $\frac{3}{4}$ of what he would gain on a hundred, or what he would gain per cent.

48. A boy bought some melons for 40 cents, and sold them for 60 cents. What did he make per cent.?

Ans. His gain was equal to half his outlay.

49. A grocer bought a lot of flour for 5 dollars a barrel; but, finding it damaged, he sold it for 4 dollars a barrel. What did he lose per cent.?

50. A man bought a share in a bank for 80 dollars, and sold it for 82 dollars. What did he gain per cent.?

51. A man bought a lot of apples for $1.50 a barrel. What must he sell them for to gain 10 per cent.?

52. A hatter bought some hats for $3.50 each. He is willing to sell them at a profit of 4 per cent. At what price will he sell them?

53. A manufacturing company declare a dividend of $7\frac{1}{2}$ per cent. What ought a stockholder to receive who owns 350 dollars in that factory?

54. A has a note against B for 140 dollars, which he sells for cash at 4 per cent. discount. What does he receive for the note?

55. A merchant buys 100 barrels of flour for 5 dollars a barrel, and sells it so as to lose 5 per cent. What does he sell it for a barrel?

He afterwards buys 250 casks of lime at 1 dollar a cask. He wishes to sell it so as to make good his loss on the flour. At what per cent. profit must he sell it, and for how much a cask?

You observe that the money invested in lime is only one half as much as was invested in flour.

56. A lends B 10 dollars for 2 months, without interest. Afterwards B lends A 5 dollars. How long can A keep it to balance the favor he did to B?

57. C lends D 100 dollars, without interest, for 4 months. Afterwards D lends C 25 dollars. How long can C keep it to balance the favor?

In these cases, you will see that the money, multiplied by the time it was kept, must, in the two cases, be equal. If 10 dollars is lent me by A, without interest, for 6 months, I can balance the favor by lending A 5 dollars for 12 months, or 4 dollars for 15 months, or 15 dollars for 4 months, or 30 dollars for 2 months, or 2 dollars for 30 months, or 20 dollars for 3 months.

58. A lends B 60 dollars for three months, without requiring interest. Afterwards B lends A 90 dollars. How long may A keep the money to balance the favor?

59. A lends B 40 dollars for three months. Afterwards B lends to A, for two months, a certain sum, the use of which should balance the favor. How large must the sum be?

60. A lends B 150 dollars for 4 months. B afterwards lends A 100 dollars. How long can A keep it to balance the favor?

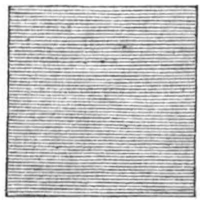
SECTION XV.

SQUARE MEASURE.

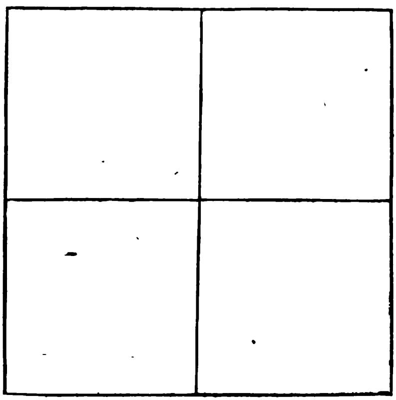
Linear measure is measure in a straight line, having length only. Square measure is the measure of surface, having length and breadth.

Thus, a linear inch,

A square inch,



*A line of 2 inches, when square, will therefore make 4 square inches; thus,



1. A line of three inches, when square, will make how many square inches?

* Note 6.

2. The square of 4 inches is how many square inches? The square of 5 inches? Of 6? Of 7? Of 8? Of 9? Of 10? Of 11? Of 12?

3. How many square inches are there in a square foot?

4. How many linear feet are there in a linear yard? How many square feet in a square yard?

5. How many square inches are there in a piece of board 12 inches long and 3 inches wide?

6. How many square inches are there in a piece of board 8 inches long and 6 inches wide? In a board 5 inches long and 3 inches wide? In a board 9 inches long and 5 inches wide?

7. How many square feet are there in the floor of a room 12 feet long and 10 feet wide?

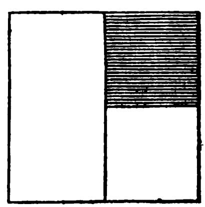
8. How many square feet are there in the floor of an entry 15 feet long and 4 feet wide?

9. How many square yards of carpeting will cover a room 6 yards long and 5 yards wide?

How many square rods are there in a piece of land 14 rods long and 8 rods wide?

10. If a road 4 rods wide passes through my land for the distance of 60 rods, how many square rods of my land does it occupy?

We will now return to the measure of an inch. If, instead of squaring a linear inch, we take only half an inch and square it, we shall have but one fourth of a square inch; thus,



So if we square one third of an inch, it will give us $\frac{1}{9}$ of a square inch. If we square one fourth of an inch, it will give us $\frac{1}{16}$ of a square inch.

11. What part of a square inch will $\frac{1}{3}$ of an inch, when squared, be? What part of a square inch will $\frac{1}{6}$ of an inch be when squared? $\frac{1}{7}$? $\frac{1}{8}$? $\frac{1}{9}$? $\frac{1}{10}$? $\frac{1}{11}$? $\frac{1}{12}$?

12. What part of a square inch is a piece of paper 1 inch long and half an inch wide? One inch long and three fourths of an inch wide?

13. What part of a square foot is a board 1 foot long and half a foot wide? One foot long and 9 inches wide?

14. How many square inches will there be in the square of a line $1\frac{1}{2}$ inches long?

[This and the following questions may be answered by drawing the figure on a slate or on a board.]

How many square inches will there be in the square of a line $2\frac{1}{2}$ inches long? $3\frac{1}{2}$? $4\frac{1}{2}$? $5\frac{1}{2}$? $6\frac{1}{2}$? $7\frac{1}{2}$? $8\frac{1}{2}$? $9\frac{1}{2}$? $10\frac{1}{2}$?

15. How many square feet in $\frac{1}{2}$ a yard squared?

16. How many square inches are there in 1 square foot?

17. How many square feet in 1 square yard?

18. How many square yards in 1 square rod?

19. How many square feet in 1 square rod?

40 sq. rods make 1 rood; 4 roods make 1 acre.

20. How many rods make 1 acre?

21. If a piece of board is 6 inches wide, how long must it be to contain a square foot?

22. If a piece of board is 3 inches wide, how long must it be to contain a square foot?

23. How long must it be to contain a square foot, if it is 2 inches wide? If 1 inch wide? If 4 inches wide?

24. If cloth is $\frac{1}{2}$ a yard wide, how much in length will make a square yard?

25. How much lining $\frac{3}{4}$ of a yard wide will line one yard of cloth one yard wide?

26. If cloth is $\frac{2}{3}$ of a yard wide, how much in length will it take for a square yard?

27. How much cloth $\frac{1}{2}$ a yard wide will it take to line 7 yards of cloth $\frac{3}{4}$ of a yard wide?

How much $\frac{3}{8}$ wide will line $1\frac{1}{2}$ yards $\frac{3}{4}$ wide?

28. How long must a strip of land 1 rod wide be to contain an acre? How long, if 2 rods wide? If 3 rods wide? If 4 rods wide? If 8 rods wide? If 10 rods wide?

29. How long must a piece of land be to contain $\frac{1}{4}$ of an acre, if it is 4 rods wide?

30. If a piece of land is 10 rods in length, how wide must it be to contain $\frac{1}{2}$ an acre?

31. A man has an acre of land 16 rods in length. How wide is it?

32. How many steps must the owner take to walk round it, if he take 5 steps to a rod?

33. A man has an acre of land 8 rods wide. How long is it? How many rods of fence will it take to fence it?

34. If a road 4 rods wide is laid out through my land, how much of the road will it take in length to make an acre? How many acres will there be in one mile of the road?

35. If it passes through my land for half a mile, and I am paid at the rate of 30 dollars an acre for the land occupied by the road, what will be the amount of damages due me?

36. If land in the city is worth 45 cents a square foot, what will be the cost of a building-lot 30 feet front and 60 feet from front to rear?

37. There are two pieces of land; one of them 12 rods square, the other 13. Which is nearest an acre?

38. There is a piece of land $12\frac{1}{2}$ rods square. How much does it fall short of an acre?

39. A painter tells me it will cost 20 cents a square yard to paint the floor of a room in my house. Supposing the room is 5 yards wide and $6\frac{1}{2}$ yards long, what will the painting of it come to?

40. What will the painting of an entry cost, at the same rate, that is $1\frac{1}{2}$ yards wide and 7 yards in length?

41. A stonecutter agrees to lay a hammered stone door-step for 50 cents for every square foot of hammered surface. The stone is 5 feet long, $3\frac{1}{2}$ feet wide, and 9 inches thick. What is the surface of the top, the two ends, and the front edge, added together? What will be the cost of the stone?

42. How many men could stand on $\frac{1}{4}$ of a mile square, allowing each man 1 square yard to stand upon?

There are various ways of finding the answer to the above questions. To encourage the student's invention, some of them will be here suggested.

First Method.—As there are $30\frac{1}{4}$ square yards in one square rod, multiply 80 (the number of rods in one fourth of a mile) by itself, and this product by $30\frac{1}{4}$. $80 \times 80 = 6400$; $6400 \times 30\frac{1}{4} = 180{,}000 + 12{,}000 + 1600 = 193{,}600$, answer.

Second Method.—Multiply 80 by $5\frac{1}{2}$, which will give the number of men in one line one fourth of a mile long. Multiply this product by itself. $80 \times 5\frac{1}{2} = 440$; $440^2 = 160{,}000 + 32{,}000 + 1600 = 193{,}600$, answer.

There are still other ways of solving the question, which the student may discover for himself.

NOTE. — *For those students who have not time to go through the book, the course of instruction in Mental Arithmetic may properly close at this place. With a similar view, the Second Part may be divided at the close of Section XXXVI. The ground thus gone over will be found to embrace all the principles and practice needed in the transactions of ordinary business. Those whose opportunities permit should have the advantage of the higher discipline furnished in the remainder of the book.*

SECTION XVI.

CONSTRUCTION OF THE SQUARE.

If I place three dots in a row, and place three such rows side by side, this will represent to the eye the square of the number 3.

Thus,

● ● ●
● ● ●
● ● ●

In the same way you may represent the square of 4, 5, or any number whatever. I will now ask your attention to the square of 4. We may make it by making a row of 4 dots, and placing 4 such rows side by side. But there is another way of coming at the square of 4. We will take the square of 3, as shown above, and see what additions we must make to it, in order to make it the square of 4. You observe that it must be wider by one row, and longer by one row, than it is now. We will then add a row above the others, and also a row on the right hand.

Thus,

* * *
● ● ● *
● ● ● *
● ● ● *

I have made the additions by stars to distinguish them from the dots. You now see there is something wanting to complete the square,—a single star in the corner.

Thus,

* * * *
● ● ● *
● ● ● *
● ● ● *

You observe, therefore, that you obtain the square of 4 by adding to the square of 3 twice 3 plus 1. We will now take the square of 4, and by additions to it obtain the square of 5. Adding a row of 4 at the top, and a row of 4 at the right hand, there will be one wanting at the corner to complete the square.

Adding this, which makes twice 4 + 1, we have the square complete. If, therefore, we have the square of any number, we can find the square of a number one greater by adding twice the first number plus 1.

The square of 5 is 25. What must you add to this square to make the square of 6?

What must you add to the square of 6 to make the square of 7? What must you add to the square of 7 to make the square of 8?

What must you add to the square of 9 to make the square of 10?

The square of 15 is 225. What is the square of 16, by the above method?

The square of 20 is 400. What is the square of 21?

The square of 30 is 900. What is the square of 31?

The square of 40 is 1600. What is the square of 41?

The square of 50 is 2500. What is the square of 51?

What is the square of 60? Of 61? Of 70? Of 71? Of 80? Of 81? Of 90? Of 91?

We will now return to the square of 3; and I ask your close attention once more. Supposing we have the square of 3 before us, and we wish to make such additions to it as shall make the square of 5. As 5 is 2 greater than 3, we must add 2 rows instead of 1. If we add 2 rows of 3 at the top, and 2 rows of 3 at the right hand, the figure will stand thus,

```
* * * ⊙ ⊙
* * * ⊙ ⊙
● ● ● * *
● ● ● * *
● ● ● * *
```

Here you see there are four stars wanting to complete the square. I have marked their places by the circle, ⊙. If you suppose these four to be added, the square will be complete, and will be the square of 5. The question is, now, What has been added to the square of 3 in order to make the square of 5? You

observe there are added 6 stars, or two rows of 3 at the top, 6 on the right hand, and 4 in the corner, to make the square of 5. But we can express this in a different way. We may consider 5 as consisting of two parts, 3 and 2 added together. We will call 3 the first part, and 2 the second part, of 5. Now, by the figure, you perceive that the square of 5 is made up, first, of the square of the first part, that is, the nine dots; then the stars at the top are the product of the first part multiplied by the second; and adding to these the stars on the right hand, we have twice the product of the first part into the second; and, last, we have, in the corner, the square of the second part.

To state it briefly once more: Regarding 5 as made up of the two parts, 3 and 2, the square of 5, we find, is equal to the square of the first part + twice the product of the two parts + the square of the second part.

This is called expressing the amount of a square in the terms of its parts.

Examine and answer the following questions:—

1. If we regard the number 6 as made up of two parts, 4 and 2, how will you express the square of 6 in the terms of its parts?

2. Regard the number 7 as consisting of two parts, 5 and 2. What is the square of 7 in the terms of its parts?

You can draw the figure for yourself, and see the application of the principle in the above cases.

It is of no consequence in what way the number is divided. The operation will bring out the exact square of the whole number in all cases. To show this, we will take the number 10, the square of which is 100. We will first divide 10 into the parts 7 and 3; then, by the formula given above, the sq. of 7 + twice the product of 7 into 3 + sq. of 3, will be the

sq. of the whole number. The sq. of $7=49$; twice $7\times3=42$; the sq. of $3=9$; $49+42+9=100$.

We will now divide 10 into the parts 6 and 4, proceeding as above. We find $36+48+16=100$.

Again; we will divide 10 into the equal parts 5 and 5. $25+50+25=100$.

Finally; divide 10 into the parts 8 and 2. $64+32+4=100$.

We will now apply the above method to the purpose of finding some squares of larger numbers.

3. What is the sq. of 25? Dividing into $20+5$; *Ans.* $400+200+25=625$.

4. What is the sq. of 35, or $30+5$? *Ans.* $900+300+25=1225$.

5. What is the sq. of 46, or $40+6$? *Ans.* $1600+480+36=2116$.

6. What is the sq. of 55? Of 64? Of 75? Of 83? Of 92?

7. What is the sq. of 125? Divide into $100+25$. 100 sq. $=10,000$; twice $100\times25=5000$; 25 sq. $=625$. *Ans.* 15,625.

8. What is the square of 150? Of 230? Of 510?

The same formula will embrace the examples mentioned in the first part of this section, when the second part of the number is 1. For example,

9. What is the sq. of 5, or $4+1$? Here twice the product of the two parts is merely twice the first part, inasmuch as multiplying by 1 does not increase the number; and the sq. of 1 is only 1. The answer, therefore, by the formula, is $16+8+1=25$.

10. This method will apply to the squaring a whole number and a fraction, as follows: What is the sq. of $1+\frac{1}{2}$? *Ans.* $1+1+\frac{1}{4}=2\frac{1}{4}$; for twice the product of $\frac{1}{2}$ into 1 is 1, and the sq. of $\frac{1}{2}$ is $\frac{1}{4}$.

To test the correctness of this answer, we will perform the operation another way. $1\frac{1}{2}=\frac{3}{2}$; $\frac{3}{2}$ sq. $=\frac{9}{4}=2\frac{1}{4}$.

11. What is the sq. of $2\frac{1}{2}$? $3\frac{1}{2}$? $4\frac{1}{2}$? $5\frac{1}{2}$? $6\frac{1}{2}$? $7\frac{1}{2}$?

The answer, in each of these cases, may be tested by changing the mixed number to an improper fraction; as, $2\frac{1}{2} = \frac{5}{2}$, &c.

We may in the same way square the sum of two fractions.

12. What is the sq. of $\frac{1}{2} + \frac{1}{2}$? *Ans.* $\frac{1}{4} + \frac{1}{2} + \frac{1}{4} = 1$. Now, $\frac{1}{2} + \frac{1}{2} = 1$, the square of which is 1.

13. What is the sq. of $\frac{3}{4} + \frac{1}{4}$? *Ans.* $\frac{9}{16} + \frac{6}{16} + \frac{1}{16} = \frac{16}{16}$ or 1. Now, $\frac{3}{4} + \frac{1}{4} = 1$, the sq. of which is 1.

14. What is the sq. of $\frac{3}{2} + \frac{1}{2}$? The sum of these is 2, the sq. of which is 4; the answer, therefore, should be 4. Applying the formula, the operation is as follows; $\frac{9}{4} + \frac{6}{4} + \frac{1}{4} = \frac{16}{4} = 4$.

PRACTICAL QUESTIONS.

We will now apply the above principle to the solution of some questions which appear at first a little difficult.

1. A boy had some apples. He placed part of them in rows making a square, and found he had 6 apples left. He placed another row on two sides, and found he had enough to complete the square except one apple at the corner. How many apples were there in the first square? How many apples had he?

2. Three boys were playing at marbles. The first says, "I have just marbles enough to make a square;" and he placed them in rows on the floor, forming a square. The second boy says, "I have twelve marbles, and I will put a row on two sides of yours, and make your square larger;" but, on placing his marbles, he found he wanted 3 more to complete the square. Then the third boy says, "I have just three, and that will make the square complete."

How many had the first boy? How large was the square which all the marbles made?

3. The boys of a school thought, one day, at

recess, they would form themselves into a square. A part of them first formed a square, when it was found that there were 15 boys left. These 15 then placed themselves in a row on two sides of the square, when it was found that it required 2 boys more to complete the square. How many boys were there in the first square? How many in all?

4. A general, drawing up his soldiers in a square body, with the same number in rank and file, found he had 55 men over and above. He placed these in a row on two sides, and found that he now wanted 30 men to complete the square. How many men were there on a side of the first square? How many men in the first square? How many men had he in all?

5. There is a certain square number expressed in the terms of its parts; that is, it is expressed in three terms, the first of which is the square of the first part, the second is twice the product of the two parts, and the third is the square of the second part. Now, the first two terms are $16+24$. What is the third term? What is the number?

6. There is a square number expressed in the terms of its parts. The first two terms are $9+24$. What is the third?

7. The first and last terms of a square are $4+25$. What must be the middle term? What is the number?

8. The first and last terms of a square are $9+4$. What is the second term? What is the square?

9. Complete the square whose first two terms are $16+40+\square$.

10. Complete the sq. $36+24+\square$.

11. Complete the sq. $4+4+\square$.

12. Complete the sq. $9+6+\square$.

13. Complete the sq. $16+8+\square$.

14. Complete the sq. $25+10+\square$.

15. Complete the sq. $25+20+\square$.

16. Complete the sq. $36+72+\square$.

17. Complete the sq. $25+30+\square$.

18. Complete the sq. $25+40+\square$.

The square root is the number which, multiplied into itself, produces the square. Thus 3 is the sq. root of 9, 2 is the sq. root of 4, 5 is the sq. root of 25.

The square root of a square of three terms, like those given above, is the square root of the first term, plus the square root of the third; for these, multiplied by themselves, will produce the square. Thus the square root of the square $9+12+4$, is $3+2$, or 5. 3 is the sq. root of 9, and 2 the sq. root of 4. This number, $3+2$, multiplied by itself, will produce the square of $9+12+4$.

19. Complete the sq. $36+60+\square$. What is its root?

20. Complete the sq. $36+12+\square$. What is its root?

21. Complete the sq. $16+4+\square$. What is its root?

22. Complete the sq. $9+12+\square$. What is its root?

23. What is the sq. root of 169?

Divide the number into three terms, $100+60+9$. We divide it so, because 100 is a square, and 60 is twice the product of 10, the root of the 1st term, into the root of what this way of dividing leaves us for the 3d term. That is, if we take 60 for the 2d term, we leave 9 for the 3d term, and this is as it should be, for 60 is twice the product of 10 into 3.

The root is therefore $10+3$, or 13.

24. What is the sq. root of 196?

We will take for the 1st term 100, whose root is 10. Now, as the 2d term is twice the product of the two terms of the root, if we divide half of it by the 1st, it will give the 2d term of the root; or, what is the same thing, if we divide the 2d term of the square by twice the 1st term of the root, it will give the 2d of the root. Now, 96 contains the 2d and 3d terms of the square. We must separate it into two parts, such

that the first part, divided by twice 10, or 20, will give for quotient the root of the second part.

Let us first try by dividing it into 60 and 36. Now, 60 divided by 20 gives 3, which is not the root of 36. Our division, therefore, was wrong. The 2d term was too small, and the 3d too great. We will try again. By dividing it into 80 and 16, we find that 80 divided by 20 gives 4, which is the root of 16. The number 196, when arranged in the three terms of the square, will be 100 + 80 + 16, and the root is 10 + 4, or 14.

25. What is the root of 225? Here we must not take for our 1st term 200, for this is not a square. We must take the largest square whose root is an even 10. This is 100. We have remaining 125. This we must divide into two terms, such that the 1st divided by twice 10 will give the root of the 2d term. We will first divide into 80 and 45. 80 divided by 20 gives 4, which is not the square root of 45. We will divide into 100 and 25. 100 divided by 20 gives 5, which is the exact root of 25. The number 225, therefore, when arranged in the three terms of a square, stands 100 + 100 + 25, and its square root is 10 + 5, or 15.

26. What is the square root of 256? Taking for the 1st term 100, it remains to divide the remainder, 156, according to the principle stated above. Now, 120 will contain 20, 6 times, which is the root of the remainder, 36. The number stands, therefore, 100 + 120 + 36; square root, 10 + 6, or 16.

27. What is the square root of 484? Here we take for the 1st term 400, for that is the largest square whose root is in even tens; its root is 20. The remainder we may divide into 80 and 4. Dividing 80 by twice 20, or 40, we have for the quotient 2, which is the root of the 3d term. The square, therefore, is 400 + 80 + 4; the root, 20 + 2, or 22.

28. What is the square root of 529? Of 576? Of 625? Of 676?

29. The following is a ready method of squaring a mixed number whose fraction is ½; as, 2½, 3½, 4½. The fraction will be ¼. For the other number, multiply the whole number by a number greater than itself by 1. Thus the square of 2½; the fraction is ¼, the whole number 2×3, or 6; 6¼. The square of 3½; 3×4, or 12, and ¼. The sq. of 4½ is 4×5, or 20, and ¼. What is the sq. of 5½? 6½? 7½? 8½? 9½? 10½?

The same principle will apply to the square of whole numbers whose last figure is 5; as, 25, 45, 55, &c.; for such a number consists of a certain number of tens, and half of 10. As the right-hand figure is 5, the two right-hand figures of the square must be 25. Then multiply the number at the left of the 5 by itself increased by 1; and this, read at the left hand of the 25, will be the square. Thus, for the square of 25, the two right-hand figures will be 25; for the rest, multiply 2 by 3, which is 6. *Ans.* 625.

30. What is the square of 35? 3×4=12. To this annex 25. 1225, *Ans.*

What is the square of 45? Of 55? Of 65? Of 75? Of 85? Of 95?

SECTION XVII.

PRACTICAL QUESTIONS IN SQUARE MEASURE.

1. How many square rods are there in a square mile?

2. How many acres are there in a square mile?

3. Divide a square mile into 4 equal farms. How many acres would there be in each?

4. How many acres are there in one fourth of a mile square?

5. How many acres are there in a town 6 miles long and 5 miles broad?

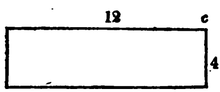
6. If half of the town is unfit for improvement, in consequence of water and mountains, how many farms of 100 acres might be made from the other half?

7. A man bought a rectangular piece of land containing 40 acres. On going out with his son to measure round it, to ascertain how much fence it would require to enclose it, they found the first side they measured to be 160 rods. "We need not measure any more," said the son, "for I can tell all the rest in my head." How wide was the piece? and how many rods of fence would it take to go round it?

8. A man bought 7 acres of land, in rectangular form. The width of it was 28 rods. What was its length?

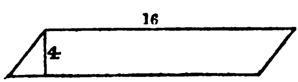
If a four-sided piece of land is rectangular, its contents may be found by multiplying two adjacent sides, or sides that meet and form a corner.

Thus, if a piece is 12 rods long and 4 rods wide,



the two boundaries, 12 and 4, which meet and form the right angle at *c*, will, when multiplied together, give the contents in square rods; $12 \times 4 = 48$.

If the opposite sides are parallel, but the angles are not right angles, the distance between the two sides must be measured by a perpendicular line, thus:

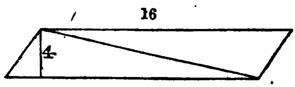


the length, 16 rods, multiplied by the width, 4 rods, will give the contents.

If the piece is a triangle, there must be a perpen-

dicular line drawn to the longest side from the angle opposite to it. This perpendicular we may call the height of the triangle, and the longest side, its length; and the height multiplied into the length, will give double the area; dividing this by 2, we get the area.

To show the reason of this, take the following figure. By examining this, you will see that there are in it two triangles just alike.



The length of each is 16 rods, and the height 4 rods. Now, 16×4 will give, as in the case above, the area of the whole figure, that is, of both the triangles; therefore it will give twice the area of one of them.

9. What is the area of a triangle whose longest side is 16 rods, and the perpendicular, from the opposite angle to this side, 12 rods?

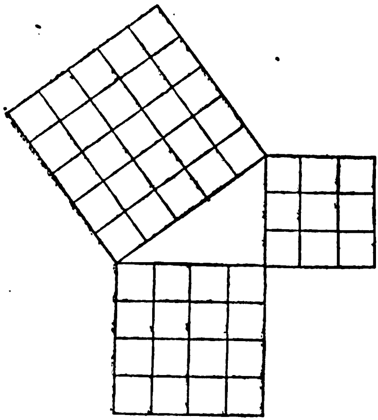
10. What is the area of a triangle whose longest side is 24 rods, and its height 9 rods?

11. What is the area of a triangle whose longest side is 18 rods, and height 16 rods?

12. A triangle contains just one acre. Its longest side is 20 rods. How long must the perpendicular be from the opposite angle to that side?

13. A triangle contains 2 acres. Its longest side is 32 rods. How long is the perpendicular, from the opposite angle to this side?

In a right-angled triangle, the longest side is called the *hypotenuse;* the sides containing the right angle are called the *legs*, or one the *base* and the other the *perpendicular*. In all right-angled triangles, the square of the hypotenuse is just equal to the sum of the squares of the two other sides. This important principle is exhibited to the eye in the following figure:—



The hypotenuse is divided into 5 equal parts, and its square is therefore 25. The base has 4 equal parts of the same length, making a square of 16. The perpendicular is divided into 3 equal parts, of the same length as the others, which makes a square of 9. The square of the perpendicular and of the base, added together, $16+9=25$, which is the square of the hypotenuse.

If we know the square of the hypotenuse, we know the sum of the squares of the two legs. If we know the sum of the squares of the two legs, we know the square of the hypotenuse. If we know the square of the hypotenuse and of one leg, we can find the square of the other leg. And if we know the square of any one of these sides, we can obtain the length of the side by extracting the square root.

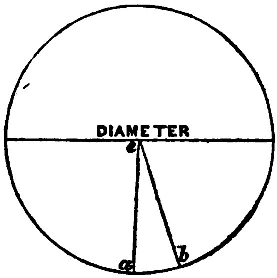
14. In a certain right-angled triangle, the square of the hypotenuse is 100 feet. What is the length of the hypotenuse?—In the same triangle, the square of the base is 64 feet. What is the length of the base?

In the same triangle, what must be the square of the perpendicular? What is the length of the perpendicular?

15. A and B set out from the same place. A travels east 6 miles. B travels north till he is 10 miles in a straight line from A. How far north has B travelled?

16. There is a triangle, the perpendicular of which is 3 feet, the hypotenuse is 5 feet. How long is the base?

17. A man had a piece of land, in the form of a right-angled triangle, the two legs of which were equal to each other, and the square of the hypotenuse was 128 rods. How many rods were there in the piece?



The circumference of a circle is 3 times and $\frac{1}{7}$ greater than the diameter. If the diameter is 1 foot, the circumference will be $3\frac{1}{7}$ feet; if the diameter is 2 feet, the circumference will be $6\frac{2}{7}$ feet.

18. If the diameter of a circle is 3 feet, what will be the circumference? If the diameter is 4 feet? If 5 feet? If 6 feet? If 7 feet?

If the diameter of a water-wheel is 16 feet, what is the circumference?

19. If the diameter of the earth is 8000 miles, what is the circumference?

To find the area of a sector of a circle, as *a*, *e*, *b*, multiply the arc by half the radius. This figure may be regarded as a triangle, the base of which is the arc, and the radius the height; and you have seen before, that, in a triangle, the base multiplied by half the height gives the area. From this we may obtain a method of obtaining the area of the whole circle.

Multiply the circumference by half the radius. For we may regard the circle as made up of a great number of small triangles, whose bases added together are the circumference of the circle, and whose height is equal to radius, being, in each case, the distance from the circumference to the centre.

20. What is the circumference of a circle whose diameter is 14 feet?

What is its area?

21. What is the circumference of a circle whose diameter is 12 feet.

What is its area?

22. What is the circumference of a circle whose diameter is 20 feet?

What is its area?

23. What is the circumference of a circle whose diameter is 28 feet?

What is the area?

SECTION XVIII.

ANALYSIS OF PROBLEMS.

1. A boy spent one half the money he had, and had 1 dollar left. How much had he at first?

2. A boy spent one third of the money he had, and had 1 dollar left. How much had he at first?

Ans. If he lost one third, he had two thirds left. If 1 dollar was two thirds, half a dollar must be one third, and $\frac{3}{2}$ of a dollar the whole. He had 1 dollar and a half.

3. A boy spent $\frac{1}{4}$ of his money, and had 1 dollar left. How much had he at first?

Let this and the following answers be given in form of a fraction, like the preceding answer.

4. A boy spent $\frac{1}{5}$ of his money, and had 1 dollar left. How much had he at first?

5. A boy spent $\frac{1}{6}$ of his money, and had 1 dollar left. How much had he at first?

6. A boy spent $\frac{1}{7}$ of his money, and had 1 dollar left. How much had he at first?

7. A boy spent $\frac{1}{8}$ of his money, and had 1 dollar left. How much had he at first?

8. A boy spent $\frac{1}{8}$ of his money, and had 1 dollar left? How much had he at first?

9. A boy spent $\frac{1}{10}$ of his money, and had 1 dollar left. How much had he at first?

10. A man carried some corn to mill. The miller took $\frac{1}{16}$ of it for toll, and then there was just a bushel. How much did the man carry to mill?

11. A man carried some cloth to be fulled. It shrank two sevenths in its length, and was then just a yard long. How long was it at first?

12. A man drew a prize in a lottery. $\frac{1}{20}$ of the prize was retained, and then the drawer received just 100 dollars. How much was the prize?

13. If a stick of timber shrink $\frac{1}{14}$ in weight in seasoning, and then weigh 100 pounds, how much did it weigh at first?

14. A teamster sold $\frac{2}{7}$ of a cord of wood, and then had half a cord left. How much had he at first?

15. A man had an estate left him by his father. He lost $\frac{4}{5}$ of it. He then received 1000 dollars, and then he had 3500 dollars. How much had he at first?

16. A merchant began trade with a sum of money, and gained so as to increase his original stock by $\frac{1}{4}$ of itself. He then lost 500 dollars, and had 4500 dollars left. How much did he begin with?

17. A man set out on a journey, and spent half the money he had for a dinner. He then paid half of what he had left for provender for his horse; then, half of what now remained for toll in crossing a bridge; and had 10 cents left. How much had he at first?

18. A boy spent $\frac{2}{3}$ of his money for a book, and $\frac{1}{7}$ of it for some paper, and had 8 cents left. How much had he at first?

19. A boy, playing at marbles, lost, in the first game, $\frac{1}{4}$ of what he had; in the second game, $\frac{1}{4}$ of what he then had; in the third, $\frac{1}{4}$ of what he then had; in the fourth, 11; and then he had 16 marbles left. How many had he at first?

20. A boy, playing at marbles, wins, in the first game, so as to double the number of marbles he had; in the second game, he loses $\frac{1}{3}$ of what he then had; in the third game, he loses 5, and then finds he has just as many as at first. How many had he at first?

21. A man had his sheep in three pens. In the first, there were 10 sheep; in the second, there were as many as in the first, and half the number in the third; in the third, there were as many as in the first and second. How many had he in all?

22. In an orchard, $\frac{1}{3}$ of the trees are plum-trees; there are 20 cherry-trees; and the apple-trees, which constitute the remainder, are half as many as the plum and cherry-trees added together. How many trees are there in the orchard?

23. John and William were talking of their ages. John says, "I am twelve years old." William says, "If half my age were multiplied by one fourth of yours, and half your age plus one subtracted from the product, that would give my age." How old was he?

24. A man, talking of the age of his two children, said the youngest was three years old; the age of the eldest was $\frac{1}{4}$ his own age; if his own age was divided by that of his youngest, and once and one third the age of the youngest subtracted from the quotient, that would give the age of the eldest. How old was the eldest?

25. The number of pupils in a school is such that, if you take half of them, and increase that by 2; then take one third of this last number, and increase it by 3; and from this number subtract 6; the remainder will be 7. How many are there in the school?

26. A boy plays three games at marbles. In the first, he loses a certain number; in the second, he gains 8; in the third, he loses 4; and then he finds he has 2 more than he began with. How many did he lose in the first game?

27. A boy, playing at marbles, first lost one third of what he had; he then doubled his number, when he had 5 marbles more than he had at first. How many had he at first?

28. There is a certain number, one third of which exceeds one fourth of it by 2. What is the number?

29. There is a certain number, one fourth of which exceeds one fifth of it by 1. What is the number?

30. There is a certain number, one third of which added to one fifth of it amounts to 16. What is the number?

31. There is a number, one third, one fourth, and one fifth of which, added together, are 94. What is the number?

32. What is that number, a fifth of which exceeds a sixth of it by 4?

33. What number is that, of which a fourth part exceeds a seventh part by 9?

34. In a certain orchard there are apple, peach, and pear-trees. The apple-trees are 2 more than half the whole. The peach-trees are one third of the whole, and are 14 less than the apple-trees. The rest are pear-trees. How many are there of each kind? and how many in all?

SECTION XIX.

SOLID MEASURE.

Whatever has length, and breadth, and thickness, is a solid. A block of wood 1 inch long, 1 inch high, and 1 inch wide, is a solid inch. A block 1 foot long, 1 foot wide, and 1 foot high, is a solid foot. A block 1 yard long, 1 yard wide, 1 yard high, is a solid yard.

1. How many solid inches are there in a block 3 in. long, 2 in. wide, and 1 in. high?

2. How many in a block 4 in. long, 3 in. wide, and 2 in. high?

3. How many solid feet are there in a block 5 feet long, 3 feet wide, and 2 feet high?

4. How many solid feet in a block 7 feet long, 2 feet wide, and 2 feet high?

When a solid has its length, height, and breadth, equal to each other, it is called a *cube;* and the linear measure of its length, height, or breadth, is called the *root of the cube.* We have seen what is a cubic inch, a cubic foot, and a cubic yard.

Suppose, now, we have a pile of cubic inch blocks, and we wish to construct from them a cube, each of whose dimensions shall be 2 inches. We will first take 2 blocks, and place them down side by side. This will be as long as the required figure, but it will not be wide enough nor high enough. To make it wide enough, we will place 2 more blocks down by the side of the former. The figure now contains 4 cubic inches, and is 2 inches long and 2 inches wide, but it is only 1 inch high. To make it 2 inches high, we must place upon this another layer of 4 blocks, arranged just like the former. The figure will then be 2 in. long, 2 in. wide, and 2 in. high; it contains 8 cubic inches, and is the cube of 2.

5. How many blocks will you require, and how will you arrange them, to make the cube of 3?

6. How many blocks will you require, and how will you arrange them, to make the cube of 4?

7. How many blocks will you require, and how will you arrange them, to make the cube of 5?

The cube, when expressed in numbers, is the same as the 3d power of the root. It is found by taking the root 3 times as a factor. Thus, the 3d power of 2 is

$2 \times 2 \times 2 = 8$. The 3d power of 3 is $3 \times 3 \times 3 = 27$; of 4, is $4 \times 4 \times 4 = 64$; of 5, is $5 \times 5 \times 5 = 125$.

In this way we may find the 3d power of any number.

8. How many blocks, of a cubic foot each, will it take to form a cubic solid 6 feet on a side?

9. How many blocks, of a cubic foot each, will it take to form a cubic solid of 7 feet each way?

10. How many cubic feet will it take to form a cube of 8 feet?

11. How many cubic feet will it take to form a cube of 9 feet?

12. How many cubic feet will it take to form a cube of 10 feet?

13. How many cubic inches are there in a cubic foot?

14. A pile of wood 8 feet long, 4 feet high, and 4 feet wide, makes a cord. How many cubic feet are there in a cord?

15. A pile of wood 4 feet long, 4 feet high, and 1 foot thick, makes what is called *a cord foot.* How many cubic feet are there in a cord foot?

16. How many cord feet are there in a cord?

17. There is a pile of wood 40 feet long, 4 feet wide, and 5 feet high. How many cords does it contain?

18. There is a stick of hewn timber 25 feet long, 1 foot wide, and 1 foot thick. How many cubic feet does it contain?

19. There is a tree, from the but-end of which a stick may be hewn 13 feet long, 2 feet wide, and 2 feet thick. How many cubic feet will it contain?

20. It is estimated that 50 feet of hewn timber weigh a ton. If 50 cubic feet weigh 20 cwt. net weight, what will 1 foot weigh?

21. If you divide a cubic inch into blocks meas-

uring $\frac{1}{2}$ an inch each way, how many such will there be in a cubic inch?

22. How many cubic half inches are in a cubic inch?

23. If you divide a cubic inch into cubes of $\frac{1}{4}$ of an inch each, how many such will there be?

24. How many cubic quarter inches are there in a cubic inch?

25. How many cubic inches are there in a cube of one inch and a half?

26. If a man digs a cellar at the rate of $\frac{3}{8}$ of a dollar for a cubic yard, what will the job come to, if the cellar is 18 feet long, 12 feet wide, and 6 feet deep?

27. A stone-layer agreed to build a solid wall 30 feet long, $4\frac{1}{2}$ feet thick, and 6 feet high, for $2\frac{1}{2}$ dollars a cubic yard. What did the wall cost?

CONSTRUCTION OF THE CUBE.

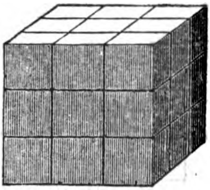
We have seen that the third power, or cube, of any number, is obtained by taking the number three times as a factor. The product is the cube, or third power.

In this way the cube of any number whatever may be obtained. There is another way, however, of constructing the cube, the knowledge of which is very important in the operation of extracting the cube root.

Suppose we wish to find the cube of 5. Instead of taking 5 three times as a factor, thus, $5\times5\times5=125$, we will regard the number 5 as consisting of two parts, 3 and 2. We will call 3 the first part, and 2 the second part, of 5.

We will begin by making the cube of the first part, 3, thus, $3\times3\times3=27$.

We will regard this as a cube of 3 inches, (that is, 3 in. long, 3 in. wide, and 3 in. high,) and represent it by the following figure:—



The question now is, How shall we enlarge this cube of 3, so as to make it the cube of 5? It is evident, it must be 2 in. longer, 2 in. broader, and 2 in. higher, than it now is. We will begin, then, by putting 2 layers of inch blocks on the front side, 2 layers on the right side, and 2 layers on the top. The figure, thus enlarged, is not a cube. There are several places not filled up. It is nearer the cube of 5 than it was before; but something more must be added. Before making that addition, however, let us see what we have done. The figure is the cube of 3, which is the first part of 5. To this there are 3 equal additions made. Each of these additions is 3 in. square, and 2 in. thick. Now, 3 is the first part of 5. Each addition, therefore, contains the square of the first part, 3, multiplied by the second part, 2 or $3^2 \times 2$. Therefore the three additions will be 3 times the square of the first part multiplied by the second.

The whole figure, therefore, after these three additions are made, contains $3^3 + 3$ times $3^2 \times 2$.

We will now see what additions must next be made to the figure.

There are 3 places that need filling up, each 3 in. long, 2 in. wide, and 2 in. high. Each of these new additions is 2 in. square and 3 in. long. It consists, therefore, of the first part of 5 multiplied by the square of the second; and the three together are 3 times the first part multiplied by the square of the second. There is one addition wanting to complete the cube; that is at the corner. It must be 2 in. long, 2 in. wide, and 2 in. high; that is, the cube of 2; or, in other words, the cube of the second part.

Remembering that the two parts of 5, as here divided, are 3 and 2; the printed figure is the cube of

the first part; the first addition is 3 times the square of the first part, multiplied by the second; the second addition is 3 times the first part, multiplied by the square of the second; the third addition is the cube of the second part. Let the letter a stand for the first part, 3; and the letter b, for the second part, 2. The printed figure will then be a^3; the first addition, $3a^2b$; the second addition, $3ab^2$; the third addition, b^3. The whole cube, therefore, will be $a^3+3a^2b+3ab^2+b^3$. Observe that the letters and numbers are to be multiplied together, though there is no sign of multiplication between them; as, $3a^2b$ is three times the square of a multiplied by b.

These are called the four terms of the cube, when the root is in two parts.

If we express the above in the numbers for the cube of 5, it will stand thus;

$$\overbrace{3^3}^{\text{1st.}}+\overbrace{3\times3^2\times2}^{\text{2d.}}+\overbrace{3\times3\times2^2}^{\text{3d.}}+\overbrace{2^3}^{\text{4th.}}.$$

1. What number makes the first term of this cube?
2. What number forms the second term?
3. What number forms the third term?
4. What number forms the fourth term?
5. What do all the four terms amount to?
6. Which of the four terms contains the third power of the first part? Which contains the second power of the first part?
7. Which contains the first power of the first part? Which term contains the first power of the second part? Which the second power? Which the third?
8. If the fourth term of the above cube were not given, how could you determine from the others what it must be?
9. If the third term were gone, how could you restore it? If the second was gone, how could you restore it?
10. If you divide the number 5 into the two parts

4 and 1, and express the cube according to the above rule, what will the first term be? What will be the second term? What will be the third term? What the fourth?

Remember, here, that all powers of 1 are 1,—neither more nor less.

Divide the number 6 into $4+2$, and form the cube according to the above rule.

11. What will the first term be? The second? The third? The fourth?

12. What will they all amount to?

Multiply 6 into itself 3 times, thus, $6\times6\times6$, and see if it amounts to the same.

13. Divide 6 into the parts $5+1$, and form the cube. What will be the first term? The second? The third? The fourth? What do they all amount to?

14. There is a cube in four terms, the first two terms of which are $3^3+3\times3^2\times1$. What must be the third term? What the fourth term? What is the number of the cube? What is the root of the cube? This root is the cube root of the first term, added to the cube root of the last.

15. There is a cube in four terms, the first two of which are $2^3+3\times2^2\times2$. What is the third term? What the fourth? What is the whole cube? What is the first part of the root? What is the second part? What is the whole root?

16. Complete the cube $4^3+3\times4^2\times2+\square+2^3$.

What is the number of the cube? What the root?

17. Complete the cube $3^3+3\times3^2\times3+3\times3\times3^2+\square$.

What is the number of the cube?

18. There is a cube in four terms, the first of which is 1000. What is the first part of the root?

19. The second term of the same cube is 300×6, or 1800. What is the third term?

20. What is the fourth term of the same cube?

21. What is the root of the above cube?

22. The first term of a cube is 1000; the second is 300×8, or 2400. What is the third term?

23. What is the fourth term of the above cube?

24. The first term of a cube is 8000; the second is 1200×3. What is the third term?

Observe that 1200 is 3 times the square of the first term; consequently, one third of it is the square of the first term.

25. What is the fourth term in the above cube? What is the root?

SECTION XX.

RATIO. — PROPORTION.

If we compare the two numbers 3 and 9 in order to ascertain their relative magnitude, we may subtract 3 from 9. We find the difference to be 6.

There is another way of comparing the two numbers. We may see how many times 3 will go in 9. We shall find the quotient to be 3.

The numbers we obtain in each of these comparisons is called the *ratio* of the two numbers; but they differ in kind. The former is called *arithmetical* ratio; the latter, *geometrical* ratio.

Arithmetical ratio, then, expresses the difference of two numbers; geometrical ratio expresses the quotient of one of the numbers divided by the other. As we shall speak only of geometrical ratio in what follows here, the word *ratio*, whenever it is used, may be understood to mean geometrical ratio. The ratio of 4 to 2, written 4 : 2, is 2; for 2 will go in 4 twice. The ratio of 12 to 3, written 12 : 3, is 4; for 3 will go in 12, 4 times.

The two numbers compared are together called the *terms of the ratio*, or simply the *ratio*. The first is called the *antecedent;* the second is called the *consequent*. These two terms, you will perceive, correspond exactly to the numerator and denominator of a fraction; for, in a fraction, the numerator is divided by the denominator. A ratio is, then, another way of expressing a fraction. The antecedent is the numerator; the consequent, the denominator. 4 : 2 is the same as $\frac{4}{2}$; 6 : 3 the same as $\frac{6}{3}$.

As a ratio is essentially the same as a fraction, every thing is true of a ratio which is true of a fraction.

1. What effect will it have on the value of the ratio, if you increase the antecedent? If you diminish the antecedent? If you double the antecedent? If you divide the antecedent by 2?

2. What effect will it have on the value of the ratio, if you increase the consequent? If you diminish the consequent? If you multiply the consequent? If you divide the consequent?

3. Take the ratio 4 : 2. How can you multiply it by 2? In what other way?

4. How can you divide it by 2? In what other way?

5. Take the ratio 6 : 3. How can you multiply it by 2? Can you do it in more than one way? If you cannot, why?

6. How can you divide it by 4? Can you do it in more than one way? If not, why?

Take the ratio 4 : 2. Multiply both terms by the same number, 3, for example. It will be 12 : 6. You see the value is not altered. — Divide both terms 4 : 2 by 2. It will be 2 : 1. The value is not altered. It is still 2.

PROPORTION.

If there are four numbers, and the first has the same ratio to the second that the third has to the fourth, the four numbers are said to be in proportion. Thus the numbers 2 : 1 : : 12 : 6 are in proportion. The first has the same ratio to the second, that the third has to the fourth. The ratio is 2.

A proportion, then, is the equality of two ratios.

The four dots : : between the two ratios, are the same as the sign of equality, =.

In order to preserve the proportion, the two ratios must always be equal to each other. You may make any change you please in the terms, provided you do not destroy this equality.

Let us take the proportion 4 : 2 : : 12 : 6. The value of the two ratios is now equal.

1st. Multiply the antecedents by 2. 8 : 2 : : 24 : 6. The numbers are still in proportion, for the value of the two ratios is equal.

2d. Divide the antecedents by 2. 2 : 2 : : 6 : 6. The value of the two ratios is equal.

3d. Multiply the consequents by 2. 4 : 4 : : 12 : 12. The value of the two ratios is equal.

4th. Divide the consequents by 2. 4 : 1 : : 12 : 3. The value of the ratios is still equal.

5th. Multiply both terms of the first ratio by 2. 8 : 4 : : 12 : 6; or multiply the 2 terms of the second ratio by 2; 4 : 2 : : 24 : 12; the ratios are still equal. In the same way we might take any other number for our operations instead of 2. The same operations might be performed without destroying the proportion.

The two middle terms of a proportion are called the *means;* the first and last terms are called the *extremes.*

In a proportion, the product of the two means is equal to the product of the two extremes.

Take the proportion 4 : 2 : : 6 : 3. The product of the means, 2×6, is 12; and the product of the extrêmes, 4×3, is 12.

Take the proportion 6 : 2 : : 9 : 3. 2×9=18, 3×6=18.

Take the proportion 10 : 2 : : 30 : 6. 2×30=6×10.

If we know, then, the product of the means, we know the product of the extremes.

In a certain proportion, the product of the means is 30. What must be the product of the extremes?

6. Further, if we know the product of the means, and if we know one of the extremes, we can find the other. If, as in the above case, the product of the means is 30, and if one of the extremes is 3, what must be the other?

How do you find that number?

7. If the product of the means is 30, and one of the extremes is 10, what must the other be?

How do you find the number?

8. If the product of the means is 30, and one of the extremes is 5, what is the other?

If one of the extremes is 6, what is the other?

If one of the extremes is 15, what is the other?

You see, therefore, that, if you multiply the means together, and divide the product by one extreme, the quotient will be the other extreme.

9. If the product of the means is 72, and one of the extremes is 24, what will the other be?

10. If the first three terms of a proportion are 9 : 6 : : 12, what must the fourth term be?

11. What is the fourth term of the proportion 5 : 3 : : 15?

12. Complete the proportion 8 : 6 : : 12.

13. Complete the proportion 14 : 8 : : 7.

14. Complete the proportion 10 : 4 : : 15.

By means of this rule, many interesting questions may be solved.

15. If 8 yards of cloth cost 6 dollars, what will 20 yards of the same cloth cost?

It is evident that the length of the shorter piece is to the length of the longer, as the cost of the shorter is to the cost of the longer. Now, we know all these numbers except the last, and can express them in the form of a proportion, thus, $\overset{\text{Yds.}}{8} : \overset{\text{Yds.}}{20} :: \overset{\text{Dols.}}{6}$. 8 is the length of the shorter piece; 20, the length of the longer; 6 is the cost of the shorter piece. The fourth term, that is, the cost of the longer piece, we have not yet found. You must discover that yourself. How can you do it?

16. If 18 yards of cloth cost 15 dollars, what will 12 yards of the same cloth cost?

What do you here seek, — the quantity of cloth, or the price? Is it the price of the longer, or of the shorter piece?

How can you make a proportion with the two quantities of cloth, and the two sums they cost? State this proportion in general terms, putting the thing sought as the fourth term. State the proportion in figures, all except the fourth term. How will you find the fourth term?

17. If 5 yards of cloth cost 2 dollars, what will 7 yards of the same cloth cost?

State the proportion in general terms.

State the first three terms in figures, and find the fourth.

18. If a horse travels 16 miles in 3 hours, how far will he travel in 2 hours?

As the longer time is to the shorter time, so is the greater distance to the smaller distance.

Remember that things of the same kind should stand in the same ratio; and that the quantity sought must be the fourth term. Then inquire what the true pro-

portion must be, and state it in general terms, repeating the trial, if necessary, till you perceive that you are right. This is far better than any special rule, for it leads you to reason on what you do.

19. If a certain number of cubic feet of timber weighs a certain number of hundred weight, and if we wish to know, without weighing, how many hundred weight a certain smaller number of cubic feet will weigh, what will be the proportion in general terms?

20. If 16 cubic feet of wood weigh 5 cwt., what will 6 cubic feet weigh?

21. If 3 barrels of flour last a family 7 months, how many barrels will last them 12 months?

22. If an iron rod, of equal size throughout, and of a certain length, weighs a certain number of pounds, and is broken into two parts, not in the middle, how can you find the weight of one of the parts without weighing it?

23. If an iron rod 7 feet long weighs 20 pounds, what will 5 feet of it weigh?

COMPARISON OF SIMILAR SURFACES.

As all the above questions may be answered by analysis as well as by proportion, the rule of proportion might be dispensed with for the purpose of solving this kind of questions. It has, however, very important and interesting applications in the measurement of similar surfaces and solids. To prepare for this, you must attend carefully to a few introductory statements.

Two surfaces are similar to each other when they are shaped alike, though they may be unequal in size. Thus a large circle is similar to a small circle, for they are both shaped alike.

So one square is similar to another, though they

may be unequal in size. One equilateral triangle is similar to another equilateral triangle.

If a rectangle is twice as long as it is wide, and a larger or a smaller rectangle is twice as long as it is wide, the two are similar. In the same way any two surfaces, however irregular their shape, are similar, provided they are shaped alike.

We will now come to a stricter definition of similar surfaces. Similar surfaces are such as have their corresponding dimensions proportional.—Take the circle. The dimensions of the circle are the diameter and the circumference. The diameter of one circle is to its circumference, as the diameter of a larger or a smaller circle is to its circumference. For you have learned before that the circumference of a circle is $3\frac{1}{7}$ times its diameter.

Take the square. One side of a square is to another side of it, as one side of a larger or smaller square is to another side of it.

If a rectangle is twice as long as it is wide, another rectangle, in order to be similar, whatever be its size, must be twice as long as it is wide.

1. There are two similar rectangles. One is 8 feet long and 6 feet wide. The other is 6 feet long. How wide is it?

2. There are two similar rectangles. One is 5 feet long and 2 feet wide. The other is 11 feet long. How wide is it?

3. There are two similar right-angled triangles. In the larger, the base is 9; the perpendicular, 4;—in the smaller, the base is 8. How long is the perpendicular?

We will now come to the comparison of the areas of similar surfaces.

4. There are two circles. One is 1 foot in diameter; the other, 2 feet. How much greater is the area of the larger than the area of the smaller?

It is clearly more than twice as large; for you could lay two of the smaller circles on the larger, and still leave a considerable space uncovered. Before answering this question, we will take the simple case of two squares, one of which measures 1 foot on a side, and the other 2 feet. You perceive the larger one is 4 times as great as the smaller.

Let one square measure 2 feet on a side; the other, 4 feet. How much greater is the larger than the smaller? The smaller contains 4 square feet; the larger, 16; it is therefore 4 times as large.

5. If one square measures 3 times as much on a side as another, how much greater is its aréa than that of the smaller?

Let one square measure 1 foot on a side; the other, 3 feet. In what ratio are their areas?—Let one measure 2 feet on a side; the other, 6. In what ratio are their areas?

The area of the larger, you find, is 9 times as great as that of the smaller. This may serve to suggest the principle by which the areas of all similar surfaces may be compared.

The areas of similar surfaces are to each other as the squares of their corresponding dimensions.

Let one square measure 1 foot on a side; another, 2 feet. $1^2 : 2^2 :: 1 : 4$.

Let one square measure 2 feet, and another, 4 feet, on a side. $2^2 : 4^2 :: 4 : 16$.

Let one square measure 1 foot on a side; another, 3 feet. $1^2 : 3^2 :: 1 : 9$.

Let one square measure 2 feet on a side; another, 6 feet. $2^2 : 6^2 :: 4 : 36$.

This principle applies to circles, triangles, and all similar surfaces whatever. You can now recur to question 4, and find the answer to it.

6. There are two circles. The diameter of the greater is 3 times that of the smaller. The area of the smaller is 1 acre. What is the area of the greater?

7. There are two circles. The diameter of the smaller is two thirds that of the greater, and the area of the smaller is 4 acres. What is the area of the greater?

8. There are two similar triangles. The corresponding dimensions are as 3 to 4, and the greater contains 12 acres. What does the smaller contain?

9. A farmer fenced a triangular piece of ground for a field; but, finding it not large enough, he enlarged it, making each side one third greater than before, and it then contained 5 acres. How much did it contain at first?

10. There is an irregular field containing 8 acres. One of the sides measures 20 rods. If the field be enlarged, retaining the same form, so that the above-named side measures 25 rods, how much land will it contain?

11. There are 2 circles. The smaller is 3 rods, the larger 7 rods, in diameter. How much greater in proportion is the area of the latter than that of the former?

12. There are 2 circles, one with a diameter of 3 feet, the other of 8. How much greater is the area of the larger than that of the smaller?

COMPARISON OF SIMILAR SOLIDS.

We now come to the comparison of similar solids.

1. Let there be 2 cubes, one of them measuring 1 inch on a side, the other 2 inches. How much greater is one than the other?

You will perceive, by thinking of the construction

12*

of the cube, that the cube measuring 2 inches has in it 8 cubic inches, and is therefore 8 times as great as the one measuring only 1 inch.

2. Take cubes measuring 1 inch and 3 inches. How much greater is the latter than the former?
3. Let one measure 1 inch, the other 4 inches. How much greater will the larger cube be?

These examples may suggest the principle on which all similar solids are compared.

Similar solids are to each other as the cubes of their corresponding dimensions.

Take, now, the first of the above three examples. The ratio of the corresponding dimensions is $2:1$; the cubes of these terms, or $2^3:1^3$, are $8:1$; and this is the proportion of the one solid to the other.

In the second example, the ratio of the corresponding dimensions is $3:1$; the cubes of these, $3^3:1^3$, are $27:1$; and this is the ratio of the two solids.

In the third example, the ratio of the corresponding dimensions is $4:1$; the cubes of these terms, $4^3:1^3$, are $64:1$, which is the ratio of the two solids to each other.

4. There are 2 iron balls. The smaller is 1 inch, the other 5 inches, in diameter. How much does the larger weigh more than the smaller?
5. There are 2 iron balls. Their diameters are 2 inches and 3 inches. What is the ratio of their weight?
6. If the diameter of 2 balls is respectively 3 inches and 4 inches, what is the ratio of their weight?
7. If a cubic inch of stone weigh 1 ounce, how many ounces would a cubic stone, measuring 10 inches, weigh?
8. How many ounces, if the cube measured 11 inches?

9. How many ounces, if the cube measured 12 inches?

10. If there were a smaller pyramid, of the same material and shape with the great pyramid of Egypt, and of $\frac{1}{10}$ its height, how many such would it take to equal in solid contents the great pyramid?

11. A common brick weighs 4 pounds, and is 8 inches in length. How much will a similarly-shaped brick weigh, that measures 16 inches in length?

12. If an axe 4 inches wide weighs $4\frac{1}{2}$ pounds, what will be the weight of a similar axe 5 inches wide?

13. If a blacksmith's anvil 1 foot long weighs 200 pounds, how much will a similar anvil weigh that is 2 feet long?

14. A farmer sells 2 stacks of hay of the same shape and solidity. The smaller is 10 feet high, and is found to weigh 3 tons. The larger is 15 feet high. How can its weight be determined, without weighing it? and what will the weight be?

15. There are 2 similar cisterns. The smaller is 6 feet deep, and holds 500 gallons. The larger is 8 feet deep. How many gallons will it contain?

These operations will be rendered more easy, if, in every case, where the ratios may be reduced, you reduce them to their lowest terms.

16. If a coal-pit 8 feet high has required 10 cords of wood, how much wood would be required for a coal-pit of similar shape 10 feet high?

17. If there are two trees shaped alike, the smaller measuring 4 feet in circumference, the larger 5 feet, how will the amount of wood in the one compare with that in the other?

The principle given above applies to all similar solids, whether bounded by plain surfaces or by curved surfaces.

18. If a dwarf measures 2 feet in height, and a man of the same form and solidity, 6 feet high, weighs 180 pounds, how many such dwarfs would equal the weight of the man? What would the dwarf weigh?

19. If a man 6 feet 2 inches in height weighs 200 pounds, what would be the weight of a giant of equal solidity and similar form, 9 feet 3 inches in height?

20. If an animal 4 feet high weighs 600 pounds, what will an animal of the same form and equal solidity weigh, whose height is 5 feet?

21. An artist in Europe has made a perfect model of St. Peter's Church at Rome, representing every part in exact proportion, on a scale of 1 foot to 100 feet. If the material of the model is of the same solidity with that of the church, how many times greater is the solid contents of the church than that of the model?

22. If a granite obelisk were constructed in the precise form of the Bunker Hill monument, of one tenth its height, how many such obelisks would the monument furnish material to construct?

The comparison of similar surfaces and solids by proportion has various interesting applications in determining the comparative strength of timbers and materials used in building and in other arts.

Case First.—The strength of materials to resist a strain lengthwise.

1. If an iron rod half an inch in diameter will hold a certain weight suspended by it, how much greater weight will a rod hold that is 1 inch in diameter?

Here the strength is in proportion to the size, without regard to the length; that is, as the square of the diameters.

2. If an iron rod half an inch in diameter will sus-

pend 2 tons, what weight will a rod suspend that is three fourths of an inch in diameter?

3. A builder finds that an iron rod 1 inch in diameter will suspend a certain weight. He wishes, however, to add to the weight half as much more, and, in order to support it, substitutes for the inch rod another rod $1\frac{1}{4}$ inches in diameter. Will it sustain the required weight?

4. There are two ropes of the same material; one, $1\frac{1}{2}$ inches in diameter; the other, 2 inches. What is the ratio of their strength?

Case Second.—The strength of beams to resist fracture crosswise. In beams of the same material, length, and width, but of different depth, the strength varies *as the square of the depth.*

1. There are two beams of equal length; but the depth of one is 10 inches; of the other, 12 inches. What is the ratio of their strength?

2. There is a stick of timber 4 inches thick and 12 inches deep. If sawed into three 4-inch joists, what part of the former strength of the whole stick, when placed edgewise, will each part possess, allowing nothing for waste in sawing?

3. There is a stick of timber 10 inches in depth. If 4 inches of its depth be removed, what will be its strength compared to what it was before?

4. There are two sticks of timber equal in length and width; one, 7 inches deep; the other, 5. What is the ratio of their strength?

5. If a stick of timber 6 inches deep have 2 inches of the depth removed, will it be weakened more than one half?

What is the exact ratio of its present, compared with its former strength?

6. A builder went to a lumber-yard, wishing to

obtain an oak beam 5 inches wide and 10 inches deep. The lumber-merchant said, "I have not such a stick; but I have two oak sticks of the right length and width, and 7 inches deep. They will both, placed side by side, be stronger than one beam 10 inches deep." "Not so strong," said the builder.

Which was right? And what is the ratio of strength in the two cases?

NOTES TO PART FIRST.

Note 1.—Page 16.

This exercise should be often reviewed, till the pupils can go through it with ease, and without mistake. No exercise can be devised that will more rapidly increase the learner's powers in addition.

Note 2.—Page 17.—*To the Instructor.*

The word *complement* means, something to fill up. In arithmetic, the complement of a number, strictly speaking, is that number which must be added to it, to make it up to the next higher order. The complement of a number consisting of units only, as 3, 7, 9, is the number that must be added to make it up to 10, and consists of units only. If the number consist of tens, as 20, 50, its complement is the number that must be added to make a hundred, and consist of tens. If the number is hundreds, its complement is so many hundreds as will make up a thousand.

If the number consist of several orders, its full complement will consist of the same orders, of such an amount as to raise the sum to the next order above the highest named in it. The complement of 745 is 255, for $745+255=1000$, which is the order next above the highest named in the given sum.

The more restricted use of the word, as employed in the text, is sufficient for the purposes here had in view.

A few suggestions will here be made in reference to the best mode of conducting the accompanying recitation. The object of the lesson is to cultivate the power of instantly associating a number and its complement together. In conducting the recitation, the answer to each question, as it is given out, should be required simultaneously of the whole

class. The teacher should stand before them, and require that every eye be fixed on him. The questions should not be hurried, but the class should be encouraged to answer instantly on hearing the question. This will be easy in the first class of numbers given, which are even tens. In regard to the remaining numbers, however, which are not even tens, something more will be necessary. Suppose the question is, What is the complement of 37? It may be conducted as follows:

Teacher. What is the complement of —— 30?

Class. 70.

Teacher. Now, listen to me without speaking. What is the complement of 30 ——? You observe, I am going to say something more. What will it be?

Class. Something between 30 and 40.

Teacher. Well, then, whereabouts will the complement be found?

Class. Between 60 and 70.

Teacher. Very good. Now, when I say 30, and keep my voice suspended, showing that that is not all, what number can you think of, that you know will be a part of the complement?

Class. 60.

Teacher. Very well. Now, listen. What is the complement of 30 ——? What have you now in your mind?

Class. 60.

Teacher. Well. Now, once more listen, and all answer as soon as you hear the question. What is the complement of 37?

Class. 63.

In the following questions, let the teacher always make a short pause between pronouncing the tens and the units; and if the class hesitate or disagree in their answer, let the question be resolved into its elements, and each one presented separately. Thus, if 64 is the number, and the class have not answered promptly and alike, say thus; "What is the complement of 60?"

Class. 40.

Teacher. What is the complement of 60 ——? What do you think of?

Class. 30.

Teacher. Now, answer all together. What is the complement of 64?

Class. 36.

In the examples of addition that follow, the teacher should make a pause between the two numbers, and see that every member of the class is intent and eager to catch the second number, and answer instantly. A few questions answered by the whole class in this way, will benefit them more than whole pages recited in an indolent and listless manner.

Note 3. — Page 17.

In these and all other examples, the large numbers should be taken first. If the pupil begins with the units, as in written arithmetic, he should be checked at once. Such a method would only lead to a laborious imitation of the process of written arithmetic, which is not the natural one, and could give no new power to the pupil, nor awaken any new interest in the study. Only a small portion of these questions should be recited at one lesson.

Note 4. — Page 25.

Care must be taken here, that the pupil does not imitate the process of written arithmetic, but be required to regard every number in its true value. Thus, in the question, "What is one fifth of 250?" he must not say, "5 in 25 is contained 5 times; and 5 in 0, no times;" but, "One fifth of 25 is 5; therefore, one fifth of 250 is 50."

Note 5. — Page 28.

In the higher as well as the lower numbers, let the pupil grapple at once with the number as it stands. In this way his interest will be very much increased. He will see, throughout, the progress he is making. Whereas, in written arithmetic, as usually studied, the pupil has no sooner begun

an operation than he loses sight of the process, and goes on in blind bondage to his rule, till he comes out at the end, and then looks to the book, as to an oracle, for the answer.

Let the oldest class in arithmetic in a school be called up, and one of them be required to perform on the board the question, "What is one sixth of 43,248?" and when he has obtained the first quotient figure, stop him, and ask him what he has now done. He will most likely be unable to tell. The answer he will give will probably be, that he has divided 43 by 6; and no one of his class will probably have a better answer to offer. If he says he has divided 43 thousand, he is still wrong; for he has divided only 42 thousand, leaving one thousand undivided.

In some of the examples given in this section, the large numbers may be separated in different ways preparatory to division. Thus, in the last example, 92,648 may be divided 80,000, 12,000, 600, 48; or 88,000, 4000, 640, 8; and in still other ways.

Pupils should be encouraged to exhibit more methods than one for obtaining the answer. If a scholar has two methods, he should be allowed to give them both; and if another has a different one still, it should be brought forward; and the most lucid and easy one should receive the commendation of the teacher.

Note 6. — Page 101.

Strictly speaking, there is no relation in quantity between a line and a surface, but only between a line and the dimensions of a surface. By the square of a line is meant a square surface, each of whose sides has the same length as the given line.

PART SECOND;

CONTAINING

RULES AND EXAMPLES FOR PRACTICE

IN

WRITTEN ARITHMETIC.

NUMERATION OF WHOLE NUMBERS.

In common arithmetic, there are 9 figures used for the expression of numbers. 1, one; 2, two; 3, three; 4, four; 5, five; 6, six; 7, seven; 8, eight; 9, nine. When one of these figures stands alone, it signifies so many units, or ones; when two figures stand side by side, the left-hand figure signifies so many tens; when three stand side by side, the left-hand figure signifies so many hundreds; and universally, as you advance to the left, the figures increase in value tenfold at each step, as will be seen in the table on the next page.

The right-hand place is always that of units. When there are tens, and no units, a cipher, 0, must stand in the unit's place, thus, 20. This merely serves to occupy the unit's place, and shows that the figure, 2, is in the place of tens. When there are hundreds, and no tens nor units, two ciphers are wanted; one in the unit's place, and one in the place of tens; as, 200; and so of all higher numbers.

To annex a cipher to a figure, therefore, is the same as to multiply the number by ten; for it removes the figure from the unit's place to the place of tens. To annex two ciphers, is the same as to multiply the number by a hundred; for it removes the figure from the unit's place to that of hundreds.

10th place, billions.	9th place, hundreds of millions.	8th place, tens of millions.	7th place, millions.	6th place, hundreds of thousands.	5th place, tens of thousands.	4th place, thousands.	3d place, hundreds.	2d place, tens.	1st place, units.	
									2	two.
								2		two tens, that is, twenty.
							2			two hundred.
						2	2	2	2	enumerate.
							2	2		enumerate.
					2					two tens of thousands, that is, twenty thousand.
				2						two hundred thousand.
			2	2						enumerate.
		2	2							twenty-two million.
2	2	2								enumerate.
2	2	2	2	2	2	2	2	2	2	enumerate.
						4	6	7	3	
					8	3				
			5	4	6					
				3	8	4				
		9	7	6	4	3	9			
	4	2	7	8	6					
						4		2	3	
					6		8	2		
				9	7		3	4	1	
				8	3	4		6		
			7	3	6	5			1	
		4	5	8	9	4	6	7	2	
			4	3	2	6	8	1	3	
		9	6	5	6	7	1	4	1	
3	1	4	7	6	2	9	3	8	1	
5	4	6	7	1	5	8	4	6	7	
6	9	1	4	5	6	3	7	9	1	
			2						1	
4	5	3	7	9	6	5	3	8	5	

13th place, trillions. 16th place, quadrillions.
19th place, quintillions. 22d place, sextillions.

In writing numbers, every place not occupied by a figure must be occupied by a cipher. Otherwise the true value of

the figures at the left hand of that place would not be preserved. Thus, if you wish to write in figures the number three hundred and four, as there are no tens, a cipher must stand in the place of tens, 304. Should you omit the cipher, and write 34, the 3 would have slid into the tens' place, and it would not express three hundred and four.

As, in advancing to the left, figures increase their value tenfold at each step, so, if you begin at any place in a line of figures, and move towards the right, the figures will diminish in value tenfold at each step; that is, each figure will signify but a tenth part of what it would, if it stood in the next left-hand place. This will prepare you to look at the

NUMERATION OF DECIMALS.

Whole Numbers.										Decimals.							
10th place, billions.	9th place, hunds. of mills.	8th place, tens of millions.	7th place, millions.	6th place, hunds. of thous.	5th place, tens of thous.	4th place, thousands.	3d place, hundreds.	2d place, tens.	1st place, units.	1st place, tenths.	2d place, hundredths.	3d place, thousandths.	4th place, tens of thou'ths.	5th place, hund. of thou'ths.	6th place, millionths.	7th place, tens of mill'ths.	
									2	2							two, and two tenths.
									4	2	5						four, and two tenths, and five hundredths, or 25 hundredths.
								2	2	2	2						twenty-two, and twenty-two hundredths.
										3	8	7					
												2					two thousandths.
							1	3	6	7	1	4					enumerate.
											3	6	1				
												4	1	7			four hundred and seventeen hundred-thousandths.
										2	2						enumerate.
										2	2	2					
										2	2	2	2				
														2			
														2	2		
										5	4	7					
											4	3					
											8	9	7				
													6	7	4	2	

SECTION I.

ADDITION.

Addition is the uniting of several sums into one, to show their amount.

Rule.—Set down the numbers, units under units, tens under tens, and so on. Add the column of units; set down the units of the amount, and carry the tens, if there are any, to the column of tens. Add the column of tens, and set down the unit figure of the amount, carrying the figure of tens to the next column; and so on. In adding the last column, set down the whole amount.

To prove the work, repeat the operation, beginning at the top, and adding downwards.

Examples.

1. 472+842.
2. 376+421+645.
3. 431+843+794.
4. 821+954+359.
5. 267+549+121.
6. 834+682+762.
7. 468+912+683.
8. 871+934+340.
9. 516+617+713.
10. 685+937+742.
11. 840+931+672.
12. 963+847+784.
13. 421+317+844.
14. 6342+1896+4741+8962.
15. 3249+856+8007+4990.
16. 3819+42+906+1728.
17. 1645+2718+92+1807.
18. 1543+1899+3054+26.
19. 1854+1962+2168+666.
20. 1062+6300+9071+7001.
21. 2593+1801+9201+2113.
22. 9064+2118+1802+3076.
23. 1001+9016+7990+26.
24. 106+2307+9436+108.
25. 1214+6403+7113+4009.

26. In 1840, the population of the New England States was as follows: Maine, 501,793; New Hampshire, 284,574; Vermont, 291,948; Massachusetts, 737,699; Connecticut, 309,978; Rhode Island, 108,850. What was the population of all the New England States?

27. The population of the Middle States, in 1840, was as follows: New York, 2,428,921; New Jersey, 373,306; Pennsylvania, 1,724,033; Delaware, 78,085; Maryland, 469,232; Virginia, 1,239,797. What was the total population of the Middle States?

28. The population of the Southern States, in 1840, was — North Carolina, 753,419; South Carolina, 594,398; Georgia, 691,392; Alabama, 590,756; Tennessee, 829,210; Mississippi, 375,651; Arkansas, 97,574; Louisiana, 352,411. What was the total population of these states?

29. In 1840, the population of the Western States was as follows: Ohio, 1,519,467; Indiana, 685,866; Illinois, 476,183; Michigan, 212,267; Kentucky, 777,828; Missouri, 383,702. What was the total population of these states?

30. What is the total population of all the United States, as set down in the four preceding examples?

Whenever, in adding a column, two figures occur together, which amount to 10, as 8 and 2, 7 and 3, take them both together, and call them 10. This will make the addition more rapid and easy.

When you have become familiar with the operations in addition, you may occasionally vary your method, by taking two columns of figures at a time. If you have been thorough in the mental part of this work, you will be able to do this. It will furnish an agreeable variation in your method of work, and greatly increase your power of rapid calculation.

31. This method is seen in the following example:

3124 7681 4942 15747	42 and 81 are 123, and 24 are 147. Set down the 47, and carry the 1 hundred to the column of hundreds. 50 and 76 are 126, and 31 are 157.

It will be well often to adopt this as your method of proof. After performing the work by taking one column at a time, prove it by taking two columns; or perform it first in the latter way, and prove it in the other.

32. 1467+894+1721+8396.
33. 9461+8134+2016+4317.
34. 84161+9632+78167+43180.
35. 109761+20671+437674+963.
36. 26431+184097+467124+84321.
37. 43126+91434+237210+127.
38. 1235467+1096+34271+4081.
39. 10467+31762+10921+9634.
40. 37193+10634+206721+104367.

SECTION II.

SUBTRACTION.

Subtraction is the taking of a smaller number from a larger, to show the difference. The larger number is called the *minuend;* the smaller, the *subtrahend;* the difference is called the *remainder.*

Rule. — Set down the numbers, the larger number uppermost, units under units, tens under tens. Subtract the units of the lower number from the unit figure above, and set down the difference. Proceed in the same way, with the tens and higher orders, to the

close. If, in any case, the figure of the minuend is less than the figure below it, increase it by 10, by borrowing 1 from the next higher figure of the minuend; remembering, at the next step, that the figure in the minuend has already been diminished by 1.

To prove the work, add the remainder and the subtrahend together, and, if the work is correct, the sum will agree with the minuend.

Examples.

1. 748—365.	14. 8990—7096.
2. 674—582.	15. 8243—6492.
3. 849—634.	16. 784—96.
4. 347—267.	17. 210—100.
5. 431—249.	18. 681—504.
6. 867—312.	19. 901—75.
7. 419—224.	20. 16432—14968.
8. 519—499.	21. 195864—137461.
9. 318—201.	22. 228476—13962.
10. 856—106.	23. 740016—116799.
11. 3416—2999.	24. 86400—199.
12. 4162—4091.	25. 10006—4364.
13. 7089—3007.	

26. America was discovered in 1492. Plymouth was settled in 1620. How long was that after the discovery of America?

27. The Independence of the United States was declared in 1776. How long was that after the settlement of Plymouth?

28. George Washington was born in 1732. He took command of the American armies in 1776. How old was he then?

29. General Washington became President of the United States in 1789. How old was he then?

30. In 1820, the population of Maine was 298,335. In 1830, it was 399,955. What was the increase in 10 years?

31. The population of Maine, in 1840, was 501,973. What was the increase from 1830 to 1840?

32. The population of Massachusetts, in 1810, was 472,040; in 1820, 523,487. How much had it increased from 1810 to 1820?

33. The population of Massachusetts, in 1830, was 610,408. How much had it increased from 1820 to 1830?

34. In 1840, the population of Massachusetts was 737,699. How much had it increased from 1830 to 1840?

35. The population of the state of New York, in 1810, was 959,949. In 1820, it was 1,372,812. What was the gain?

36. The population of New York, in 1830, was 1,918,608. What was the gain from 1820 to 1830?

37. In 1840, it was 2,428,921. What was the gain from 1830 to 1840?

38. The population of Ohio, in 1810, was 230,760. In 1820, it was 581,434. What was the gain from 1810 to 1820?

39. In 1830, the population of Ohio was 937,903. What was the increase from 1820 to 1830?

40. In 1840, the population of Ohio was 1,519,467. What was the increase from 1830 to 1840?

Another method of performing subtraction, often more convenient than the former, is the following:—

Regard the subtrahend as a round number, one greater than the figure of its highest order; that is, if the subtrahend is 43, call it 50; if 251, call it 300. Subtract this round number from the minuend, and then to the remainder add the complement required to make up the subtrahend to the round number; as follows:—

41. 674—381. 400 from 674 leaves 274. Add 19, the complement of 381. $274+19=293$, *Ans.*

Apply this method to example 11, above.

42. 3416—2999. The first remainder, you see, is 416, and the complement is 1. *Ans.* 417.

This example shows how much shorter the work often becomes by adopting this method.

One of the above methods may be used as a proof of the other.

43. 384—219.
44. 1260—984.
45. 1679—291.
46. 2496—954.

SECTION III.

MULTIPLICATION.

In multiplication, a number is repeated a certain number of times, and the result thus obtained is called the *product*.

Rule.—Set down the smaller factor under the larger, units under units, tens under tens. Begin with the unit figure of the multiplier. Multiply by it, first, the units of the multiplicand, setting down the units of the product, and reserving the tens to be added to the next product. Proceed thus through all the figures of the multiplicand. If there are more figures than one in the multiplier, take, next, the tens, and multiply the figures of the multiplicand as before, setting the figures of the product one degree farther to the left than before.

Add the several partial products, and the amount will be the whole product.

Examples.

1. 342 × 64. Thus,

```
    342
     64
   ----
   1368
  2052
  -----
Ans. 21888
```

Proof. The best practical method of proof is carefully to repeat the operation.

2.	346 × 34.	20.	13763 × 26.
3.	579 × 82.	21.	97623 × 318.
4.	976 × 38.	22.	1172671 × 216.
5.	826 × 91.	23.	1874215 × 341.
6.	376 × 121.	24.	742634 × 912.
7.	345 × 243.	25.	189423 × 62.
8.	798 × 114.	26.	14376281 × 194.
9.	6181 × 35.	27.	17284265 × 36.
10.	6821 × 82.	28.	671234 × 427.
11.	7413 × 96.	29.	1895453 × 28.
12.	7921 × 22.	30.	3469528 × 672.
13.	8964 × 85.	31.	906421384 × 923.
14.	9056 × 43.	32.	713489605 × 84.
15.	8007 × 41.	33.	843469537 × 906.
16.	4559 × 741.	34.	236749024 × 516.
17.	9642 × 864.	35.	443754262 × 916.
18.	8721 × 317.	36.	1123496113 × 413.
19.	1841 × 134.		

37. The average length of the state of Massachusetts is 150 miles; its breadth, 50 miles. How many square miles does it contain?

38. The average length of Pennsylvania is 275 miles; its breadth, 165 miles. How many square miles does it contain?

39. The state of Ohio averages 223 miles in length, 180 in breadth. How many square miles does it contain?

40. The state of Illinois averages 245 miles in length, 147 in breadth. How many square miles does it contain?

41. If there are 365 days in one year, how many days are there in 25 years?

42. If the wages of a soldier is 8 dollars a month, what will be the wages of 7867 soldiers for 12 months?

43. There are 320 rods in 1 mile. How many rods are there in 278 miles?

44. 741×84.	47. 946734×496.
45. 19643×892.	48. 1623×198.
46. 246731×9210.	49. 9336×1998.

When the multiplier is a composite number, you may multiply first by one of its factors, and the product thus obtained by the other factor, or by the others in succession, if there are more than two.

Apply this method to the following examples:—

50. 8476×45. 51. 1371×125. 52. 7465×108.

If a figure in the multiplier is a factor of the figure in the next higher place, you may shorten the operation by multiplying the partial product of the lower figure by the other factor of the higher. Thus, in example 44, above, having found 4 times 741, you know that 8 times the same is twice as many, and 80 times is 20 times as many. You need, therefore, only double the line of the first partial product, setting it one degree farther to the left, to express the tenfold higher value. The same may be done if the right-hand figure is a factor of the number expressed by the next two higher figures.

Apply the process to the following examples:—

53. 947×639.	56. 27934×369.
54. 13674×4812.	57. 67514×64164.
55. 19742×568.	58. 259385×13212.

If the multiplier is 10, or any power of 10, annex

to the multiplicand, for the answer, as many ciphers as there are in the multiplier.

If the multiplier consists of 9's, add as many ciphers to the multiplicand as there are 9's in the multiplier, and from the product subtract the multiplicand. The remainder will be the product sought; for, by adding the ciphers, you multiply by a number greater by one than the multiplier. The multiplicand, therefore, will be found in the product once too many times. So, if the multiplier is 2 or 3 less than some power of 10, you may do the same, remembering to take the multiplicand out as many times as the multiplier is units less than a power of 10.

In this way perform the following examples:—

59. 3847×99.
60. 4572×999.
61. 54327×98.
62. 45314×997.

SECTION IV.

DIVISION.

In division, two numbers are given, in order to find how many times one contains the other, or in order to separate one number into as many equal parts as there are units in the other.

The number to be divided is the *dividend;* the number it is divided by is the *divisor;* the answer is the *quotient.*

To perform the operation, set down the divisor at the left of the dividend. Take as many figures on the left of the dividend as will contain the divisor one or more times. See how many times the divisor is contained in these figures, and set down the number as the first figure of the quotient. Multiply the

divisor by the quotient figure, and subtract the product from the number taken. To the remainder bring down another figure of the dividend; and proceed as before.

1. 13276 ÷ 122; thus,

```
122 ) 13276 ( 108, quotient.
      122
      ----
       1076
        976
       ----
        100, remainder.
```

To prove the work, multiply the divisor and the quotient together, and add the remainder, if there be any; and the amount, if the work be right, will be equal to the dividend.

Thus, in the above example,

```
  122
  108
  ---
  976
 122
  100
-----
13276
```

2. 11764 ÷ 34.
3. 47478 ÷ 82.
4. 37088 ÷ 38.
5. 75116 ÷ 91.
6. 45496 ÷ 121.
7. 83835 ÷ 243.
8. 90972 ÷ 114.
9. 89743 ÷ 17.
10. 7426831 ÷ 141.
11. 3059 ÷ 214.
12. 700601 ÷ 34.
13. 643817 ÷ 150.
14. 300796 ÷ 145.
15. 3264291 ÷ 27.
16. 18947633 ÷ 181.
17. 384628910 ÷ 26.
18. 137900 ÷ 62.
19. 3946908 ÷ 172.

20. A man divided 35,785 dollars equally among five children. How much did each receive?

21. In one barrel of flour there are 196 lbs. How many barrels of flour are there in 13,916 lbs.?

22. In 1840, the population of Maine was 501,793.

The state contained then 30,000 square miles. How many inhabitants were there, on an average, to a square mile?

23. The state of Massachusetts contained, in 1840, 737,699 inhabitants. Its territory is 7500 square miles. How many inhabitants are there to a square mile?

24. The population of Ohio, in 1840, was 1,519,467. Its territory is 40,000 square miles. How many inhabitants to a square mile?

If the divisor is less than 12, the multiplication and subtraction may be carried on in the mind, and only the quotient set down. This may most conveniently be written directly under the dividend.

25. $7846 \div 3$. Operation, $3\,)\underline{7846}$

$2615 + 1$, remainder.

26. $964385 \div 5$.	31. $146710063 \div 6$.
27. $346218 \div 7$.	32. $1143762 \div 11$.
28. $214681 \div 9$.	33. $1964217 \div 12$.
29. $684219 \div 8$.	34. $4691382 \div 4$.
30. $9640279 \div 4$.	

It is well to adopt the method of short division sometimes when the divisor is larger than 12.

35. $33467 \div 15$.	38. $91674 \div 21$.
36. $46943 \div 15$.	39. $673845 \div 22$.
37. $81743 \div 16$.	

Miscellaneous Examples on the foregoing Rules.

1. A merchant began to trade with 4325 dollars. He gained in one year 784 dollars. What was he then worth?

2. A man's income is 948 dollars a year. His expenses are 762 dollars. How much does he save of his income in one year?

3. How much will he save in 9 years?

4. A man bequeathed his property, 3882 dollars, one third to his wife, and the remainder, in equal shares, to his four children. What was each child's share?

5. A merchant buys 643 barrels of flour at 5 dollars a barrel. He pays in addition, for freight, 65 dollars; for insurance, 17 dollars. What does the whole cost him then? What does each barrel cost him?

6. A drover bought 7 oxen for 46 dollars a head, 12 cows for 32 dollars a head, 96 sheep for 3 dollars a head. How much did they all come to?

7. A drover buys 48 head of cattle at 32 dollars a head. The whole expense of driving them to market and selling them is 72 dollars. He sells them for 38 dollars a head. What does he gain?

8. A laborer receives 16 dollars for every four weeks' labor. He works 48 weeks. What do his earnings amount to?

9. A man buys 7 tons of hay in the field for 13 dollars a ton. The cost of carrying it all to market is 48 dollars. He sells it for 15 dollars a ton. Does he gain, or lose? and how much?

10. A man receives a salary of 950 dollars. He spends for groceries 154 dollars; for milk, 21 dollars; for meat, 75 dollars; for wood, 67 dollars; for clothing, 184 dollars; for horse-hire, 38 dollars; for journeying, 93 dollars; for repairs, 19 dollars; for hired help, 132 dollars; for attendance of the physician, 26 dollars; for furniture, 51 dollars; for house rent, 184 dollars; and 86 dollars in charity and other incidental expenses. Has he spent more than his salary, or less? and how much?

11. A man works five months for 23 dollars a month, and boards himself. He pays for 24 weeks' board at 2 dollars a week. He expends, besides, for a hat, 3 dollars; for boots, 4 dollars; and for other articles of clothing, 6 dollars. How much does he

lay up, deducting the above expenses from the amount of his earnings?

12. A ship sails from Boston with a crew of 19 men. At New Orleans, 3 of the crew are discharged, and 7 new hands taken on board. The ship then sails to Liverpool, where 5 of the crew desert, 2 are left on account of sickness, and 3 new hands taken. At Havre, the ship's next port, 4 men desert, and 3 new hands are taken, when the ship sails for Boston. What is the number of her crew on the return? and what do the whole wages of the men amount to, reckoning the time from Boston to New Orleans one month; from New Orleans to Liverpool, one month and a half; from Liverpool to Havre, half a month; from Havre home, one month and a quarter; allowing 9 dollars a month to each man as far as Liverpool, and 8 dollars a month for the remainder of the voyage?

13. What is the sum of 3 times 694 divided by 2; 9 times 1836 divided by 27; and 14 times 923?

14. What is the amount of the following bill of provisions, namely, 18 barrels of beef, at 6 dollars a barrel; 19 hundred weight of ham, at 8 dollars a hundred weight; 173 barrels of flour, at 5 dollars a barrel; and 73 bushels of rye, at 92 cents a bushel?

15. The area of Pennsylvania is 46,000 square miles. If the population is 1,740,000, how many does that give, on an average, to each square mile? How many to a township containing 30 square miles?

16. New York has the same area as Pennsylvania If the population is 2,440,000, how many is that to a square mile? How many to a township containing 30 square miles?

17. Virginia has an area of 64,000 square miles. If the population is 1,240,000, how many are there to a square mile? How many, on an average, to a county containing 420 square miles?

18. How much greater was the increase of population in Massachusetts, (see Section II.) from 1820 to

SECTION XII.

DIVISIBILITY OF NUMBERS.

In order to ascertain if a number is divisible by either of the following numbers, 2, 3, 4, 5, 6, 8, 9, 10, or any combination of these, see Sec. VIII., Part I.

To ascertain if a number is divisible by any other number than the above, make trial of other prime divisors, as 7, 11, 13, 17, &c., beginning with the smallest, till you find one that will divide the given number, or find that it is indivisible.

Remember, that, in making trial by these numbers, you need not go higher than the square root of the given number; for, if a number is divisible, one of the factors will certainly be as small as the square root. Let us take the number 1079. What are its prime factors? By inspection you may see it is not divisible by 2, 3, 5, or 11, consequently not by 4, 6, 8, 9, 10, or 12. On trying it by 7, it is found not divisible by 7. The next number is 13. This divides it, giving a quotient, 83, which is prime. Its only factors, therefore, are 13 and 83.

Examples.

1. What are the prime factors of 667?
2. What are the prime factors of 406?
3. What are the prime factors of 419? Of 361? Of 742? Of 281? Of 316?
4. Prime factors of 941? 812? 749? 1116? 246? 8104?
5. Prime factors of 266? 884? 1917? 376?

SECTION XIII.

REDUCTION OF FRACTIONS.

[See Section VIII., Part I.]

1. Reduce $\frac{48}{96}$ to its lowest terms. *Ans.* $\frac{1}{2}$.
2. Reduce $\frac{36}{40}$ to its lowest terms.
3. Reduce $\frac{96}{100}$ to its lowest terms.
4. Reduce $\frac{40}{72}$ to its lowest terms.
5. Reduce $\frac{99}{108}$ to its lowest terms.
6. Reduce $\frac{187}{221}$ to its lowest terms. In this example, it is not evident on inspection whether the two terms of the fraction have any common divisor. In such cases, you may adopt the following

Rule to find the Greatest Common Divisor.

Divide the greater number by the less; and then take the divisor for a new dividend, and divide it by the remainder; and so on, till there is no remainder. The last divisor will be the greatest common divisor.

Apply the above rule to the sixth example.

```
187)221(1
    187
    ---
     34)187(5
        170
        ---
         17)34(2
            34
            --
            00
```

The greatest common divisor is, therefore, 17; and, dividing the terms of the fraction by this, we have for the lowest terms, $\frac{11}{13}$.

Demonstration of the Rule.

If the larger number is a multiple of the smaller, it is evident that the smaller is a common divisor of the two numbers. It is also the greatest common divisor;

for a number cannot be divided by any number greater than itself. The answer, therefore, is found by the first division. But if there is a remainder, next find whether the remainder will exactly divide the divisor. If it will, it will divide both the original numbers; for, if it will divide the divisor, it will divide any multiple of the divisor; and, as it will of course divide itself, it will divide any multiple of the divisor plus itself. Now, the larger of the original numbers is a certain multiple of the smaller plus the remainder. If, therefore, after the first division, the remainder will divide the divisor, it is a common divisor, or measure, of the two numbers.

It is also the *greatest* common divisor; for, as it will exactly measure the smaller of the two numbers, it will exactly measure any multiple of the smaller. Now, the greater number is a certain multiple of the smaller plus the remainder. The remainder, therefore, in measuring the larger number, is obliged to measure itself. No number greater than itself can do this. Therefore the remainder is the *greatest* common divisor. If the work has to be carried on farther than the second division, the same reasoning in the demonstration will apply.

Examples.

7. What is the greatest common divisor of 874 and 437?

8. What is the greatest common divisor of 497 and 451?

9. What is the greatest common divisor of 817 and 913?

10. What is the greatest common divisor of 1007 and 1219?

11. What is the greatest common divisor of 608 and 192?

12. What is the greatest common divisor of 869 and 1343?

When there are more than two numbers, first find the greatest common divisor of two of them, and then of that divisor and the third number.

13. What is the greatest common divisor of 608, 941, and 451?

Whenever it is possible, by inspection, to separate the numbers into their prime factors, this method should be adopted.

14. What is the greatest common divisor of 94, 804, and 126?

15. What is the greatest common divisor of 1274, 896, and 580?

Apply the above rules to the reduction of the following fractions:—

16. Reduce $\frac{375}{561}$ to its lowest terms.

17. Reduce to their lowest terms, $\frac{663}{867}$; $\frac{198}{432}$; $\frac{1326}{3468}$.

18. Reduce to their lowest terms, $\frac{253}{299}$; $\frac{188}{235}$; $\frac{301}{344}$.

19. Reduce to their lowest terms, $\frac{943}{1189}$; $\frac{603}{2010}$; $\frac{498}{913}$.

To reduce an Improper Fraction to a Whole or Mixed Number.

Perform the division indicated by the fraction as far as possible. If there is a remainder, express that part of the division by placing the denominator under the remainder.

20. Reduce $\frac{17}{13}$ to a whole or mixed number. *Ans.* $1\frac{4}{13}$.

21. Reduce $\frac{27}{8}$ to a whole or mixed number. *Ans.* $3\frac{3}{8}$.

22. Reduce to a whole or mixed number, $\frac{39}{8}$; $\frac{45}{17}$; $\frac{39}{11}$.

23. Reduce to a whole or mixed number, $\frac{59}{8}$; $\frac{79}{12}$; $\frac{118}{45}$.

24. Reduce the improper fractions, $\frac{119}{42}$; $\frac{780}{7}$; $\frac{341}{11}$; $\frac{144}{8}$.

Examples.

1. 24.5+68.3+17.14+87.96+3.125.
2. 165.3+96.45+8.431+.641+9412.5.
3. 450.61+27.134+89.4216+.984.
4. 64.25+3.125+87.25+181.7.
5. 125.17+34.27+.125+3761.5.

6. 186.4—27.31; 800.4—21.67.
7. 34.21—18.525; 94.31—81.167.
8. 167.51—35.125; 204.5—31.09.
9. 20.41—3.817; 601.4—517.24.
10. 648.62—.541; 346.4—91.324.
11. 5.1—1.324; .5—.0067.
12. .81—.126; .94—.3816.

Multiplication and Division.

Rule.—Perform the operation as in whole numbers; and, in inserting the point, remember that every product or dividend has as many decimal places as both the factors which produce it.

13. 124.3×87; 321.67×24.3.
14. 97.125×6; 31.4×.125.
15. 37.5×.94; 18.4×64.
16. 21×.106; 312×.05.
17. 31.1×.004; 18.61×.03.
18. 641×.41; 843.5×.95.
19. 184.2×.121; 35.6×.025.
20. .625×71; .875×31.5.

21. 84÷.012; 965÷.15.
22. 1.65÷15; 846÷3.4.
23. 1640÷.96; 425÷.055.
24. 1÷.001; 2÷.0002.
25. .001÷2; 384÷.0012.
26. 96÷.024; 64÷.016.
27. 1827÷.9; 34÷.17.

28. $.63 \div 8$; $.15 \div 14$.
29. $.48 \div .9$; $.33 \div 16$.
30. $181 \div .41$; $41 \div .6$.
31. $35 \div .36$; $48 \div .47$.
32. $.17 \div 31$; $.26 \div .013$.
33. $43 \div .06$; $45 \div .003$.
34. $75 \div .125$; $.95 \div .04$.
35. $.18 \div .0045$; $11 \div .34$.
36. $9 \div .0225$; $.7 \div .035$.
37. $80 \div .18$; $51 \div .031$.
38. $.55 \times .031$; $71.4 \times .13$.
39. $8.4 \div .021$; $.65 \div .8$.
40. $1.21 \times .09$; $.14 \times .03$.
41. $.64 \times .31$; $.08 \times .009$.
42. $36 \div .13$; $28 \div 11.4$.
43. $40.1 \div 8$; $64 \div .9$.
44. $81.4 \div .03$; $7 \div .4$.
45. $9 \div .5$; $15 \div .7$.
46. $80.2 \times .03$; $16 \div .9$.
47. $105.4 \div 37.15$.
48. $118.75 \div .0044$.

SECTION XXII.

REDUCTION OF VULGAR FRACTIONS TO DECIMALS.

[See Section XIII., Part I.]

Examples.

1. Reduce $\frac{3}{8}$ to a decimal.

Rule. — Reduce the numerator to tenths, and divide the result by the denominator, pointing off as required

in division of decimals. If there is a remainder, reduce it to the next lower order, and divide again.

2. Reduce $\frac{5}{8}$ to a decimal.
3. Reduce to decimals, $\frac{1}{8}$; $\frac{7}{8}$.
4. Reduce to decimals, $\frac{1}{16}$; $\frac{3}{16}$; $\frac{5}{16}$.
5 Reduce to decimals, $\frac{9}{16}$; $\frac{11}{16}$; $\frac{15}{16}$.

If the fractions are reducible to decimals without a remainder, obtain the answer exactly. If they are irreducible, obtain the proximate answer to four places, and annex the fractional remainder. In order to know if a fraction is exactly expressible in decimals, see Section XIII., Part I., as directed above.

6. Reduce to decimals, $\frac{19}{32}$; $\frac{23}{32}$; $\frac{27}{32}$.
7. Reduce to decimals, $\frac{4}{27}$; $\frac{9}{31}$; $\frac{5}{34}$.
8. Reduce to decimals, $\frac{11}{18}$; $\frac{21}{147}$; $\frac{3}{174}$.
9. Reduce to decimals, $\frac{21}{431}$; $\frac{172}{647}$; $\frac{31}{33}$.
10. Reduce to decimals, $\frac{1}{782}$; $\frac{4}{119}$; $\frac{5}{631}$.

In ordinary transactions, it is usual to carry the decimal answer to three or four places. The remainder is then so small in value, that it may be dropped as of no importance. At whatever place you stop, however, the decimal obtained, and the fractional remainder, when added together, will exactly equal the original fraction.

11. In order to show this, we will take $\frac{1}{7}$. Reducing it, $\begin{array}{r} 7\,)\,\underline{10} \\ 1+\frac{3}{7}, \end{array}$ we obtain, at the first step, 1 tenth $+\frac{3}{7}$ of 1 tenth. Adding these, $\frac{7}{70}+\frac{3}{70}=\frac{10}{70}=\frac{1}{7}$, which is the original fraction.

We now carry the reduction one step farther; $\begin{array}{r} 7\,)\,\underline{100} \\ 14+\frac{2}{7}. \end{array}$ We obtain 14 hundredths $+\frac{2}{7}$ of a hundredth. Adding these, $\frac{98}{700}+\frac{2}{700}=\frac{100}{700}=\frac{1}{7}$, the original fraction.

We will carry the reduction one step farther;

$$7\overline{)1000}$$
$$142+\tfrac{6}{7}.$$

We obtain 142 thousandths $+\frac{6}{7}$ of a thousandth. Adding these, by using the common denominator 7000, $\frac{994}{7000}+\frac{6}{7000}=\frac{1000}{7000}=\frac{1}{7}$, the original fraction.

12. Reduce $\frac{2}{11}$ to a decimal of one figure, with the remainder; carried to 2 places, with the remainder; carried to 3 places, with the remainder.
13. Reduce $\frac{4}{19}$ to a decimal of 7 places.
14. Reduce $\frac{7}{13}$ to a decimal of 9 places.
15. Reduce $\frac{8}{21}$ to a decimal of 10 places.

Repeating and Circulating Decimals.

When a fraction is irreducible, the decimal figure will either repeat, as, $\frac{1}{3}=.333+$; or the decimal figures obtained by the partial reduction will, after a time, recur again, in the same order as at first. Thus, $\frac{1}{11}$ gives .090909+, and so on, without end. When the same figure is repeated continually, it is called a *repeating decimal;* when the same series of different figures recurs, it is called a *circulating decimal.*

SECTION XXIII.

REDUCTION OF DENOMINATE INTEGERS TO DECIMALS.

1. Reduce 5s. 11d. to the decimal of a £.

First, reduce the quantity to the vulgar fraction of a £. Then reduce that vulgar fraction to a decimal.

2. Reduce 3s. 2½d. to the decimal of a £.

3. Reduce 5d. to the decimal of a guinea.
4. Reduce 3 qts. to the decimal of a bushel.
5. Reduce $2\frac{1}{2}$ pints to the decimal of a gallon.
6. Reduce 3 feet 5 inches to the decimal of a rod.
7. Reduce 7 feet 8 inches to the decimal of a rod.
8. Reduce 15 rods $9\frac{1}{2}$ feet to the decimal of a furlong.
9. Reduce 23 rods 13 feet to the decimal of a mile.
10. Reduce 5 hours 18 minutes to the decimal of a day.
11. Reduce 21 hours 6 minutes to the decimal of a week.
12. Reduce $12\frac{1}{2}$ square rods to the decimal of an acre.

SECTION XXIV.

TO FIND THE INTEGRAL VALUE OF DENOMINATE DECIMALS.

1. What is the value of .7 of a rod?

Supposing the quantity was 7 rods, its value in feet would be found by multiplying it by $16\frac{1}{2}$; $16\frac{1}{2}\times7=115\frac{1}{2}$, or 115.5. But it was not 7 rods, but 7 tenths of a rod, whose value we wish to find. The answer obtained, therefore, is 10 times too large. Dividing by 10, it is $11.55=11$ feet and 55 hundredths. In order to find the value in inches of 55 hundredths of a foot, we will call it 55 feet; the answer is, $55\times12=660=$ 660 feet. But, as we regarded the 55 as 100 times greater in value than it is, the answer is 100 times too large. Dividing it by 100, the answer is 6.60 inches, $=6$ inches and 60 hundredths or 6 tenths.

The above analysis shows the nature of the operation in all cases.

2. What is the value, in feet and inches, of .3 of a rod?
3. What is the value of .94 of a rod?
4. What is the value of .26 of a rod?
5. How many shillings and pence are there in .65 of a £?
6. How many shillings and pence are there in .8 of a £?
7. How many pence are there in .7 of a shilling?
8. How many pence are there in .16 of a shilling?
9. What is the value of .19 of a £?
10. What is the value of .74 of a bushel?
11. What is the value of .9 of a bushel?
12. What is the value, in rods and feet, of .7 of an acre?
13. What is the value of .9 of an acre?
14. What is the value of .12 of an hour?
15. How many minutes and seconds in .15 of an hour?
16. Find the value of .34 of a week.
17. Find the value of .162 of a week.
18. Find the value of .84 of a minute.
19. How many feet in .761 of a cord?
20. How many feet and inches in .2 of a cord?
21. How many feet in .74 of a cord?
22. How many feet in .13 of a cord?

SECTION XXV.

PRACTICAL EXAMPLES.

1. Add \$1.50+\$.375+\$.0625+\$.1875+\$5.00.
2. Add \$34.75+\$6.00+\$.375+\$.08.
3. A man had \$50, and spent \$.375 of it. How much had he left?

4. A man had $10.00, and spent $.875 of it. How much had he left?

5. A watch cost $45.675; the chain and key, $4.845. What did the whole cost?

6. The owner then sold the watch, chain, and key, for $48.375. How much did he lose?

7. A man set out on a journey with $10.00. The first day, he spent $1.125. How much had he left?

8. The second day, he spent $1.425. How much had he left?

9. The third day, he spent $1.67. How much had he left?

10. The fourth day, he spent $.875. How much had he left?

11. What is the cost of 21 lbs. of flour at $.05 per pound?

Why do you point off two decimals in the answer?

12. What is the cost of 35 lbs. of flour at $.045 per pound?

Why do you point off three decimals?

13. What is the cost of 12.5 lbs. of flour at $.05 a pound?

14. What is the cost of 15.5 lbs. of flour at $.045 a pound?

15. What is the cost of 26.25 lbs. of flour at $.0375 a pound?

16. What is the cost of 13.75 lbs. of flour at $.0425 a pound?

17. What is the cost of 15 barrels of flour at $4.75 a barrel?

18. What is the cost of 17.5 barrels of flour at $5.25 a barrel?

19. What is the cost of 3 tons of hay at $7.56 a ton?

20. What is the cost of 13.5 tons of hay at $9.00 a ton?

21. What are 17 barrels of cider worth at $1.75 a barrel?

22. What cost 16 gallons of molasses at \$.345 a gallon?

23. Divide \$1.05 into 21 equal parts. What will each part be?

24. How many pounds of flour will \$15.75 buy, at \$.045 a pound?

25. How many times is \$.05 contained in \$.625?

26. How many pounds of flour can be bought for \$.6975 at \$.045 per pound?

27. How many times is \$.0375 contained in \$984.375?

28. How many times is \$.0425 contained in \$584.375?

29. How many barrels of flour will \$71.25 buy, at \$4.75 per barrel?

30. How many barrels of flour will \$91.875 buy, at \$5.25 per barrel?

31. How many tons of hay can be bought for \$22.68, at \$7.56 per ton?

32. How many times is \$9.00 contained in \$121.50?

33. A shipmaster paid \$29.75 for ballast, giving \$1.75 a ton. How many tons did he buy?

34. How many times is \$.345 contained in \$5.52?

35. What cost 14 lbs. of flour at \$.045 a pound, and 28 lbs. of sugar at \$.095 a pound?

SECTION XXVI.

PRACTICAL QUESTIONS IN VULGAR AND DECIMAL FRACTIONS.

1. Bought 7 cwt. 15 lbs. sugar at \$6.62½ per cwt., and sold it at 7 cents per pound. What was the gain?

2. Bought 156 gallons of wine at 93 cents per gallon, and sold it at 34 cents per quart. What was the gain?

3. Bought 7 cwt. 1 qr. 11 lbs. coffee at $12.50 per cwt., and sold it at 14 cents per pound. What gain?

4. Bought 37 yards broadcloth at $5.25 per yard. Sold 20 yards of it at $7.00 per yard, and the remainder at $6.31 per yard. What was the gain?

5. Bought 24 yards broadcloth at $6.40 per yard. Sold 22¾ yards at $7.25 per yard, and the remnant for 5 dollars. What was the gain?

6. Bought 87 E. e. calico at 17 cents per E. e., and sold it at 21 cents per yard. What gain?

7. Bought 4 dozen books at $1.50 per dozen, and sold them at 16 cents each. What gain?

8. Bought 13 dozen brooms at $1.04 per dozen, and sold them at 15 cents each. What gain?

9. Bought 5½ dozen mats at $3.40 per dozen, and sold them at 36 cents each. What gain?

10. Bought 17 bushels of salt at 65 cents per bushel, and sold it at 21 cents per peck. What gain?

11. Bought one barrel of wine at 78 cents per gallon, and sold it at 16 cents per pint. What gain?

12. Bought 3 dozen baskets at $2.05 per dozen, and sold 1 dozen at 31 cents, 1 dozen at 37 cents, and 1 dozen at 42 cents each. What gain?

13. Bought 48 yards broadcloth at $5.62 per yard. Lost 17 yards by fire, and sold the remainder at $6.25 per yard. How much gain or loss?

14. Bought a hogshead molasses, containing 131 gallons, at 34 cents per gallon. 16 gallons leaked out. Sold the remainder at 37 cents per gallon. What gain or loss?

15. Bought 3½ dozen axes at $6.80 per dozen, and sold them at 92 cents each. What gain?

16. Bought 7 dozen pails at $1.42 per dozen, and sold them at 21 cents each. What gain?

17. Bought $8\frac{1}{2}$ dozen shovels at \$9.25 per dozen, and sold them at \$1.00 each. What gain?

18. Bought 74 yards carpeting at 73 cents per yard, and sold it at $87\frac{1}{2}$ cents per yard. What gain?

19. Bought 164 bushels corn at 54 cents per bushel. Sold 93 bushels at 67 cents, and the remainder at 50 cents, per bushel. How much loss or gain?

20. Bought 75 barrels apples at \$1.37 per barrel. Lost 15 barrels by decay, and sold what remained at \$2.12 per barrel. What loss or gain?

21. Bought 13 dozen oranges at 7 cents per dozen. Lost by decay $2\frac{1}{2}$ dozen, and sold the remainder at $2\frac{1}{2}$ cents each. What gain?

22. Bought 15 dozen pairs of shoes at \$4.87 per dozen, and sold them at 63 cts. per pair. What gain?

23. Bought $18\frac{1}{8}$ thousand of boards at \$9.50 per thousand. Sold 6 thousand at \$12.25 per thousand, and the remainder at \$8.42 per thousand. What gain?

24. Bought $21\frac{1}{2}$ cords wood at \$4.75 per cord. Sold 8 cords at \$5.50 per cord, and the remainder at \$4.25 per cord. What gain or loss?

25. Bought 209 bushels apples at 27 cents per bushel. Sold 46 bushels at 49 cents per bushel, and the remainder at 25 cents per bushel. What gain?

SECTION XXVII.

REDUCTION OF CURRENCIES.

English Currency.

1. Reduce 67£ to dollars and cents.

As 4s. 6d. or 54d. = \$1.00, (see Table, p. 40,) and 20s. or 240d. = 1£, 1 dollar is $\frac{54}{240}$ of a £. Reducing

this fraction to its lowest terms, it is $\frac{9}{40}$. The question, therefore, is this: In 67£ how many $\frac{9}{40}$ of a £? Dividing 67 by the fraction, we have $297\frac{7}{9}$ dollars for the answer. The fraction $\frac{7}{9}$ gives 77 cents and 7 mills.

2. Reduce 87£ to dollars and cents.
3. Reduce 104£ to dollars and cents.
4. Reduce 64£ to dollars and cents.
5. Reduce 167£ to dollars and cents.
6. Reduce 520£ to dollars and cents.
7. Reduce 84£ 6s. to dollars and cents.

First reduce the 6s. to the decimal of a £; $\frac{6}{20}=.3$. The sum then is, 84.3£. Reduce it in the same way as the cases above.

8. Reduce 124£ 13s. to Federal money.
9. Reduce 36£ 9s. 6d. to Federal money.
10. Reduce 71£ 18s. 4d. to Federal money.

To Reduce Federal Money to Sterling.

11. In 684 dollars how many pounds, shillings, and pence?

As $1.00 = $\frac{9}{40}$ of a £, 1£ = $\frac{40}{9}$ of $1.00. The question therefore is, In 684 dollars how many $\frac{40}{9}$ of $1.00? Dividing by the fraction, we have for the answer, £153.9, or 153£ 18s.

12. In $74.25 how many pounds, shillings, and pence?
13. Reduce $186.40 to Sterling money.
14. Reduce $564.35 to Sterling money.
15. Reduce $640.15 to Sterling money.

The comparative value of the dollar and the pound sterling, as given above, is called the *nominal par value.* The actual value of the pound is higher than is here given. This difference is usually estimated in

trade by adopting the nominal par value, given above, as the basis of the calculation, and then adding or subtracting a certain per cent., as 8 or 10 per cent., to compensate for the inequality of value.

Canada Currency.

5 s. = 60 d. = $1.00.

16. In 74 £ 15 s. how many dollars and cents?

As $1.00 = 60 d., and 1 £ = 240 d. $1.00 is $\frac{60}{240}$, or $\frac{1}{4}$ of a pound; multiplying by 4, the answer is $299.00.

17. In £ 126 12 s. Canada currency how many dollars and cents?

18. Reduce $841.50 to Canada currency.

New England Currency.

6 s. = 72 d. = $1.00.

19. In 64 £ 8 s. how many dollars and cents?

$1.00 = $\frac{72}{240} = \frac{3}{10}$ of a £. Reduce the 8 s. to a decimal of a pound, and divide by the fraction; we have 214.66\frac{2}{3}$.

20. Reduce 120 £ 12 s. 6 d. to Federal money.

New York Currency.

8 s. = 96 d. = $1.00.

21. Reduce 146 £ 6 s. 4 d. to Federal money.

As $1.00 = $\frac{96}{240} = \frac{4}{10}$ of a pound, reducing the shillings and pence to the decimal of a pound, and dividing by the fraction, we have $365.75.

22. Reduce 54 £ 10 s. 6 d. to Federal money.

Pennsylvania Currency.

7 s. 6 d. = 90 d. = $1.00.

23. Reduce 16 £ 5 s. 6 d. to Federal money.

$1.00 = $\frac{90}{240} = \frac{3}{8}$ of a £.

24. Reduce 7 £ 8 s. 9 d. to Federal money.

Miscellaneous Examples.

25. Reduce 187£ 8s. sterling to Federal money.
26. Reduce 964£ 16s. sterling to Federal money.
27. Reduce 1643 dollars to Sterling money.
28. Reduce 1600 dollars to Sterling money.
29. Reduce 167£ 14s. Canada, to Federal money.
30. Reduce $196.50 to Canada currency.
31. Reduce $1674.40 to New England currency.
32. Reduce $744.15 to New York currency.
33. Reduce 142£ 14s. 6d. New York, to New England currency.
34. Reduce 643£ 15s. 9d. Pennsylvania currency to Federal money.
35. Reduce $172.31 to Pennsylvania currency.

SECTION XXVIII.

INTEREST.

[See Section XIV., Part I.]

Rule.—Find the interest of 1 dollar for the given time; multiply the principal by it, and point off as in the multiplication of decimals.

1. What is the interest of $156.34 for 11 months and 20 days?

As the interest of 1 dollar for 2 months is 1 cent, for 10 months it will be 5 cents, .05. As the interest of 1 dollar for 6 days is 1 mill, for 30 days it will be 5 mills, and for 20 days 3 mills and $\frac{1}{3}$, making 8 mills and $\frac{1}{3}$. Set down the 8 at the right hand of the .05, and for the $\frac{1}{3}$ divide by 3.

```
3)156.34
    .058
--------
  125072
 78170
    5211
--------
$9.11983, Ans.
```

Observe, the 0 before the 5 must be retained; otherwise it would be 5 tenths of a dollar, or 50 cents, and the answer would be 10 times too great. If there are no cents, there must be two ciphers at the left hand of the mills. The number of cents for the multiplier is always equal to half the greatest even number of months; the number of mills is one sixth of all the days over and above the greatest even number of months.

2. Interest of $384.18 for 7 months and 10 days?
3. Interest of $147.19 for 5 months 15 days?
4. Interest of $568.25 for 9 months 13 days?
5. Interest of $81.40 for 10 months 14 days?
6. Interest of $56.32 for 12 months 24 days?
7. Interest of $75.30 for 14 months 18 days?
8. Interest of $644.46 for 15 months 24 days?
9. Interest of $831.00 for 1 year, 4 months, 12 days?
10. Interest of $380.00 for 1 year 7 months?
11. Interest of $500.00 for 1 year, 5 months, 6 days?
12. Interest of $27.42 for 4 months 17 days?
13. Interest of $13.18 for 6 months 23 days?
14. Interest of $1000.00 for 5 months 4 days?
15. Interest of $65.48 for 30 days, or 1 month?
16. Interest of $94.00 for 30 days?
17. Interest of $840.60 for 18 days?
18. Interest of $632.00 for 18 days?
19. Interest of $349.40 for 12 days?
20. Interest of $267.62 for 12 days?
21. Interest of $384.92 for 15 days?
22. Interest of $811.19 for 20 days?
23. Interest of $673.94 for 5 months 11 days?
24. Interest of $460.00 for 8 months 18 days?
25. Interest of $460.00 for 8 months 18 days, at 12 per cent.?
26. Interest of $460.00 for 8 months 18 days, at 8 per cent.?

27. Interest of $460.00 for 8 months 18 days, at 7 per cent.?

28. Interest of $460.00 for 8 months 18 days, at 5 per cent.?

29. Interest of $1500.00 for 15 months, at 4 per cent.?

30. Interest of $145.80 for 7 months 11 days, at 8 per cent.?

31. Interest of $341.18 for 2 years, 9 months, 18 days?

As the interest of a dollar for 30 days is 5 mills, for $\frac{1}{5}$ of 30 days, or 6 days, it is 1 mill. As 1 mill is $\frac{1}{1000}$, one thousandth of a dollar, it follows that the interest of 1 dollar for 1 day is one sixth of a thousandth, or $\frac{1}{6000}$ of a dollar. For two days, therefore, it will be $\frac{2}{6000}$, for 15 days $\frac{15}{6000}$, of a dollar.

A convenient rule, therefore, when the time is short, is the following: —

Multiply the sum by the number of days, and divide the product by 6000.

This is often the shortest method. You divide by 1000, by removing the decimal point three places to the left. It only remains, then, after doing this, to multiply by the number of days, and divide by 6.

32. What is the interest of $348.25 for 18 days?

Dividing by 1000, you have 0.348\frac{1}{4}$, thirty-four cents eight mills and a quarter. Instead, now, of multiplying by 18 and dividing by 6, you may multiply by 3, for 18 is 3 times 6.

3 times 0.348$\frac{1}{4}$ is 1.044\frac{3}{4}$, *Ans.*

33. Interest of $725.80 for 24 days?

34. Interest of $341.18 for 36 days?

35. Interest of $67.45 for 54 days?

36. Interest of $641.18 for 42 days?

37. Interest of $84.16 for 15 days?

To find the amount, add the interest to the principal; or, find the amount of $1.00 for the given time, and multiply the principal by it.

38. What is the amount of $560.50 for 8 months 12 days?
39. Amount of $964.25 for 15 months 18 days?
40. Amount of $460.00 for 1 year 6 months?
41. Amount of $120.50 for 2 years 4 months?
42. Amount of $68.40 for 1 year, 6 months, 24 days?
43. Amount of $500.00 for 2 years 3 months?
44. Amount of $730.50 for 6 months 12 days?
45. Amount of $840.25 for 4 months 18 days?
46. Amount of $40.50 for 8 months 12 days?

SECTION XXIX.

PARTIAL PAYMENTS.

When partial payments are made on a note, the amount due on the final payment of the note may be found by the following rule:—

Find the interest on the note up to the time of the first payment. If the payment exceeds the interest, deduct it from the amount, regarding the remainder as a new principal. On this, calculate the interest to the time of the next payment; and so on. If any payment is less than the interest then due, reserve it, and compute the interest on to the time when the payments, added together, shall exceed the interest due. Then subtract the sum of the payments from the amount then due, and proceed as before.

1. A note of 200 dollars is given July 1, 1834, on which are the following partial payments: —

Dec. 15, 1834, . . $25.00.
March 1, 1835, ... 2.50.
Aug. 10, 1835, . . . 45.00.

What was due Dec. 31, 1835?

2. A note of $340.25 is given Aug. 1, 1840.

Endorsements, — Jan. 10, 1841, . . $28.40.
July 1, 1841, ... 9.00.
March 14, 1842, . . 74.00.

What was due Jan. 1, 1843?

3. A note of $480.00 is given June 9, 1841.

Endorsements, — Sept. 11, 1842, . $60.00.
Jan. 3, 1843, ... 95.00.
March 12, 1844, . 100.00.

What was due Dec. 1, 1844?

4. A note of $675.40 is given July 3, 1843.

Endorsements, — Jan. 4, 1844, . . . $65.00.
April 17, 1844, . . 29.50.
Nov. 18, 1844, . . . 74.00.

What is due Jan. 1, 1845?

5. A note of $345.40 is given April 1, 1843.

Endorsements, — Dec. 1, 1843, . . . $40.00.
June 10, 1844, . . . 90.00.
Oct. 4, 1844, ... 31.50.
Feb. 6, 1845, ... 17.00.

What is due Jan. 1, 1846?

The rule given above is the legal rule. When, however, the note is paid within a year from the time when it was given, the following rule is usually employed: —

Find the amount of principal and interest of the whole note, from the time it was given till the final

payment; find the amount of each payment, from the time it was paid till the final payment; and the sum of these amounts subtract from the amount of the whole note. The remainder will be the balance due.

6. A note of $525.00 is given Sept. 1, 1844.

Endorsements, — Dec. 30, 1844, . . $58.75.
March 4, 1845, . . 104.20.
June 8, 1845, ... 63.40.

What is due Aug. 21, 1845?

7. A note of $784.50, given July 7, 1844, has the following endorsements: —

Sept. 5, 1844, . . . $54.00.
Nov. 10, 1844, . . . 60.00.
Jan. 12, 1845, ... 75.00.
March 17, 1845, . 100.00.

What is due May 1, 1845?

ANNUAL INTEREST.

When a note is given payable at a longer period than a year from the date, it is usual to express in the note that the interest shall be paid annually. At the end of a year, the holder of the note may compel the payment of the interest. In such cases, the debtor, instead of paying the interest that is due, sometimes renews the note, adding the interest to the principal. Thus, at the end of each year, the interest due is added in, and goes to make a new principal for the following year. This is called *Compound Interest;* but the computation of it is the same as in simple interest; for, if the interest is not computed every year, and either paid or put into the note by renewal, that interest cannot draw interest.* The law regards it as

* In some of the states, the interest, after it falls due, draws simple interest till it is paid.

the duty of the creditor to remind the debtor of his debt, by exacting the payment of the interest every year. If he does not do this, he can derive no advantage from the promise in the note to pay the interest annually.

ILLUSTRATION.

$100.00. Boston, *March* 1, 1845.

For value received, I promise to pay to John Jones, or order, one hundred dollars, in five years, with interest annually.

Samuel Barton.

If John Jones does not exact the interest till the end of the five years, and if he obtains no renewal of it, the amount of the note will be only $130.00; for the interest of 100 dollars, for five years, is 30 dollars.

If, however, he obtains a renewal of the note at the end of each year, the principal of the note, for the second year, will be $106.00.

8. What will the principal of the note for the third year be?

9. What will the principal of the note for the fourth year be?

10. What will the principal of the note for the fifth year be?

11. What will be due, principal and interest, at the end of the fifth year?

12. How much would the holder of the above note lose by omitting to obtain any renewal of it, or any payment of annual interest?

$250.00. New York, *July* 1, 1845.

For value received, I promise to pay John Foss, or order, two hundred and fifty dollars, in four years, with interest annually.

Amos Carr.

13. If no interest is paid on this note till the principal is due, and if no renewal of the note is made, what will be the amount of the note at the time of payment?

14. If the note is renewed each year, what will be the principal of the note for the second year?

15. What will be the principal of the note for the third year?

16. What will be the principal of the note for the fourth year?

17. What will be the amount of the last note at the time of payment?

18. How much would the holder, John Foss, lose, by neglecting to obtain any annual payment of interest, or any renewal of the above note?

SECTION XXX.

DISCOUNT.

[See Section XIV., Part I.]

Examples.

1. What is the present worth of $475.50, payable in 3 months?

Rule.—Deduct from the given sum its interest for the specified time. The remainder will be the present worth.

2. What is the present worth of $341.00, payable in 65 days?

3. Present worth of $940.25, payable in 4 months?

4. Present worth of $156.30, payable in 96 days?

5. Present worth of $312.60, payable in 35 days?

6. Present worth of $500.00, payable in 41 days?
7. Present worth of $814.67, payable in 65 days?
8. Present worth of $46.30, payable in 20 days?
9. Present worth of $124.45, payable in 5 months?
10. Present worth of $360.20, payable in $4\frac{1}{2}$ months?

SECTION XXXI.

BANKING.

[See Section XIV., Part I.]

To find the present worth of a note given to a bank, payable at some future time, find the present worth of 1 dollar for the given time, and multiply the sum named in the note by it.

1. What is the present worth of a note for 100 dollars, discounted at a bank, for 60 days?

Interest of 1 dollar for 63 days is .0105. This subtracted from 1 dollar, leaves for the present worth .9895.

2. What is the present worth of a note for $450.00, discounted at a bank, for 90 days?
3. I give my note to a bank for $250.00, for 60 days. What do I receive?
4. I give my note to a bank for $520.00, for 120 days. What do I receive?
5. Present worth of a bank note for $600.00, discounted for 60 days?
6. Present worth of a bank note for $150.00, discounted for 120 days?
7. Present worth of a bank note for $75.00, discounted for 30 days?

8. Present worth of a bank note for $1000.00, discounted for 60 days?

9. Present worth of a bank note for $560.00, discounted for 120 days?

10. Present worth of a bank note for $150.00, discounted for 30 days?

To find what must be the face of a note given to a bank, in order to obtain a certain sum,—*Find the present worth of* 1 *dollar for the given time, and divide the sum you wish to obtain by it. The quotient will express the sum that must be named in the note.* This, you observe, is just the reverse of the preceding case.

11. For what sum must I give my note to a bank, payable in 60 days, in order to receive $98.95?

12. For what sum must I give my note to a bank, payable in 120 days, in order to receive $509.34?

13. For what sum must I give my note to a bank, payable in 60 days, in order to receive $593.70?

14. For what sum must I give my note to a bank, payable in 30 days, in order to receive $10000.00?

SECTION XXXII.

LOSS AND GAIN.—PER CENTAGE.

[See Section XIV., Part I.]

1. A man bought a horse for 75 dollars, and sold him for $82.50. What did he gain per cent.?

2. A man bought a chaise for $178.00, and sold it for $154.50. What did he lose per cent.?

3. A merchant bought a lot of flour at $4.62 a

barrel, and sold it at $5.15 a barrel. What was his gain per cent.?

4. A merchant bought a piece of broadcloth for $4.30 per yard. What must he sell it for to gain 12 per cent.?

5. A man has $1200.00 invested in a manufactory. He receives, for his half-yearly dividend, 30 dollars. What per cent. is that on his stock?

6. A merchant fails, owing $8540.00, and can pay but $2700.00. How much will that be on a dollar?

7. A man, failing in business, agrees to pay his creditors 87 cents on a dollar. What must a creditor receive whose claim is $740.30?

The pupil should be encouraged habitually to reason upon the operations he performs; so that his method of procedure may be suggested by the relations of the numbers, and not dictated by a special rule. To aid in this important habit, a few remarks will be made on some of the foregoing examples. These may serve as specimens of analysis, and suggest to the student a similar course of reasoning in other cases.

Example 1. The whole gain is $7.50. If this gain were made on an outlay of 1 dollar, the gain would be seven hundred and fifty per cent. But the gain is made on an outlay of 75 dollars. The gain per cent., therefore, is one seventy-fifth of the whole gain.

Example 4. If the cost was 1 dollar a yard, he must add 12 cents; if 2 dollars, he must add 24 cents; &c.

Example 5. If 30 dollars had been the gain upon 1 dollar, it would have been 30 hundred per cent. But the gain was upon 1200 dollars. The per cent., therefore, must be one twelve-hundredth of 30 dollars.

8. I invest in a factory 1260 dollars, and receive for my yearly dividend 86 dollars. What is that per cent.?

9. I purchase flour at $4.75 per barrel. What must I sell it for to gain 12 per cent.?

10. A merchant bought a ship for 11475 dollars, and sold her for $13680. What did he gain per cent.?

11. The population of the state of New York, in 1810, was 959949. In 1820, it was 1372812. What was the gain per cent. in that term of 10 years?

12. In 1830, it was 1918604. What was the gain per cent. from 1820 to 1830?

13. In 1840, it was 2428921. What was the gain per cent. from 1830 to 1840?

14. The population of Ohio, in 1810, was 230760. In 1820, it was 581434. What was the gain per cent.?

15. The population of Ohio, in 1830, was 937903. What was the gain per cent. from 1820 to 1830?

16. In 1840, it was 1519467. What was the gain per cent. from 1830 to 1840?

17. Massachusetts had, in 1810, 472040 inhabitants. In 1820, it had 523287. What was the gain per cent. in 10 years?

18. Massachusetts had, in 1830, 610408 inhabitants. What was her gain per cent. from 1820 to 1830?

19. In 1840, Massachusetts had 737699 inhabitants. What was the gain per cent. from 1830 to 1840?

20. An agent sells 12000 dollars' worth of cloth for a factory, charging 2½ per cent. commission. What will be his remuneration?

21. If I buy for a merchant, at a commission of 4 per cent., 500 barrels of flour, at $4.40 per barrel, what am I entitled to for my commission?

22. What is 3 per cent. on $674.54?

23. What is 2 per cent. on $781.50?

24. What is the value of five 100 dollar shares in a bank, at 4½ per cent. advance?

25. What is the value of seven 100 dollar shares, at 6 per cent. discount?

26. What is the value of 18 shares bank stock, 60 dollars a share, at 4 per cent. discount?

27. What is the duty on a quantity of broadcloth, whose value is 1735 dollars, at 15 per cent.?

28. What is the duty on a quantity of iron, whose value is 3456 dollars, at 18 per cent.?

29. What is the commission on the sale of 1246 dollars' worth of cloth, at 3 per cent.?

30. A man bought a lot of hay for 13 dollars a ton. He sold it for $14.25 a ton. What did he gain per cent.?

31. Bought tea for 46 cents a pound. What must I sell it for a pound to gain 12 per cent.?

32. What is the worth of 750 dollars, bank stock, at 7½ per cent. advance?

33. What is the worth of 8500 dollars, bank stock, at 9 per cent. discount?

34. I sell flour at $5.32 per barrel, and thereby gain 12 per cent. on my outlay. What did the flour cost?

Every $1.00 laid out in the purchase has brought me a return of $1.12. The number of dollars I paid out on a barrel must therefore equal the number of times $1.12 will go in $5.32.

35. A merchant sells a ship for 13680 dollars, gaining thereby $14\frac{4}{10}$ per cent. on what she cost him. What did the ship cost?

36. 300 dollars is 2½ per cent. on what sum?

37. $15.63 is 2 per cent. on what sum?

38. Bought 12 barrels of flour, each containing 196 pounds, at $5.42 per barrel, and sold it at 26 cents for 7 pounds. How much gain in the whole, and how much gain per cent.?

39. Bought 43 dozen pairs of shoes, at $4.30 per dozen, and sold them at 62 cents per pair. What gain in all? What gain per cent.?

40. Bought 20 barrels of apples, each containing $2\frac{3}{8}$ bushels, at \$2.10 per barrel, and sold them at \$1.25 per bushel. What gain in all? What gain per cent.?

41. Bought 375 barrels of flour, at \$5.20 per barrel, and sold 200 barrels at \$6.10; the remainder at \$6.42 per barrel. What gain in all? What gain per cent.?

42. Bought 34 acres of land, at 41 dollars per acre. Sold it for \$1700.00. How much gain in all? What gain per cent.?

SECTION XXXIII.

ALLIGATION.*

The operations under this rule show the method of finding the value of a mixture, when the price and quantity of each of its ingredients are given; also, to find the quantity of each ingredient, when its price is given, and it is required to unite them so as to form a mixture of a given value.

CASE 1.—*To find the value of the mixture, when the quantity and price of each of the ingredients are given.*

1. Mix 15 bushels of oats, at 40 cents per bushel; 12 bushels of barley, at 60 cents; and 24 bushels of

* The word *alligation* signifies *a tying together*, and has reference to a particular way of linking numbers together, by means of which operations of this kind have been performed. The name is retained as a matter of convenience; but I have thought it best for the progress of the pupil, that he should pursue a strictly analytical method in all the operations.

corn, at 83 cents. What will the mixture be worth per bushel?

It is evident that, if you find the value of the whole, and divide the sum by the number of bushels, the quotient will be the value per bushel.

2. Mix 20 pounds of tea, at 43 cents per pound; 18 lbs. at 61 cents; and 11 lbs. at 74 cents per pound. What will the mixture be worth?

3. If 41 lbs. of coffee, at 13 cents per lb., be mixed with 45 lbs. at 9½ cents, and 27 lbs. at 15 cents, what will the mixture be worth per pound?

CASE 2. — *To find the quantity of each ingredient, when its price and that of the required mixture are given.*

4. If I mix oats, worth 2s. per bushel, with rye, worth 5s., so as to make the mixture worth 3s. per bushel, in what proportion must I mix them?

It is evident, that, if I put in 1 bushel of oats, I gain 1 shilling. Now, I must put in rye enough with this bushel of oats to lose 1 shilling. On every bushel of rye put in, I lose 2 shillings; therefore, in order to lose 1 shilling, I must put in ½ a bushel. I must therefore put in 1 bushel of oats to ½ a bushel of rye. It is evident that, if I double the quantity thus found of each ingredient, the value of the mixture will be the same; or I may take any equal multiples of the quantities, as 4 bushels of oats and 2 bushels of rye, 6 bushels of oats and 3 bushels of rye, 20 bushels of oats and 10 bushels of rye, &c.

5. If I mix oats, worth 2s. per bushel, with rye, worth 6s., so as to make the mixture worth 3s. per bushel, in what proportion must they be mixed?

6. Mix oats, worth 3s. per bushel, with wheat, worth 7s., so as to make the mixture worth 5s. per bushel. In what proportion must they be mixed?

7. Mix the same ingredients, at the same price, so as to make the mixture worth 6s. per bushel. In what proportion must they be mixed?

8. In what proportion must oats, worth 2s., and wheat, worth 8s., be mixed, to make the mixture worth 4s. per bushel?

9. How can you mix corn, worth 80 cents per bushel, and rye, worth 85 cents, with barley, worth 46 cents, so as to make a mixture worth 60 cents per bushel?

Here you have three ingredients. First, mix barley with one of the dearer ingredients, so as to make a mixture of the required value. Then mix barley with the other ingredient, and see how much you have taken of each.

10. Mix 3 sorts of tea, at 25 cents, 33 cents, and 40 cents, per pound, so as to make a mixture worth 30 cents per pound.

11. Mix tea at 20 cents, with tea at 45 cents, and tea at 54 cents, per pound, so as to make a mixture worth 38 cents per pound.

12. If you mix sugar, at 6 cents, 8 cents, 10 cents, and 11 cents, per pound, in what quantities may they be taken so as to make a mixture worth 9 cents per pound?

First, take two of the ingredients, one cheaper and one dearer than the mixture. Form a mixture of these. Then take the two remaining ingredients in the same way.

13. If three sorts of spirit, worth 60 cents, 75 cents, and 80 cents, per gallon, are mixed with water, costing nothing, what must be the proportion to make a mixture worth 70 cents per gallon?

It is immaterial in what way you select the pairs of ingredients, provided, in each pair, one of the ingredients be cheaper and the other dearer than the

required mixture. Thus a great variety of answers may be obtained whenever there is more than one pair of ingredients. In all cases, however, the correctness of the operation may be proved in the following way: —

Find the total value of all the ingredients. If this is equal to the value of the whole mixture at the required price, the work is right.

14. Mix 5 sorts of grain, at 25 cents, 30 cents, 33 cents, 45 cents, and 50 cents, so as to make a mixture worth 40 cents, per bushel.

Case 3. — *When the quantity of one ingredient is given.*

15. Mix brandy, at 74 cents per gallon, with 24 gallons of brandy, at 1 dollar per gallon, so that the mixture may be worth 80 cents per gallon.

Here you observe that the quantity of one of the ingredients is given. We will first make a mixture of the two, without regard to this circumstance. If I put in 1 gallon at 1 dollar, I lose 20 cents. For every gallon at 74 cents, which is put in, I gain 6 cents. In order to gain 20 cents, I must, therefore, put in $3\frac{1}{3}$ gallons. The quantities stand, then, 1 gallon at 1 dollar, $3\frac{1}{3}$ gallons at 74 cents. But I wish to put in 24 gallons at 1 dollar. To balance this, I must therefore put in 24 times $3\frac{1}{3}$ gallons at 74 cents; that is, 80 gallons.

16. Mix sugar, at 8 cents, 11 cents, and 12 cents, with 100 lbs. of sugar at 7 cents, so as to make the mixture worth 10 cents, per pound.

Case 4. — *When the quantity of the required mixture is given.*

17. Mix oats, at 40 cents, with corn, at 60 cents, so

as to form a mixture of 100 bushels, worth 48 cents per bushel.

If I put in 1 bushel at 40 cents, I gain 8 cents; if I put in 1 bushel at 60 cents, I lose 12 cents. To lose 8 cents, therefore, I must put in only $\frac{2}{3}$ of a bushel. The quantities are, then, 1 bushel at 40 cents, $\frac{2}{3}$ of a bushel at 60 cents; making, when added, $1\frac{2}{3}$ bushels. But 100 bushels is the quantity required. $100 \div \frac{5}{3} = 60$. Each ingredient, therefore, must be multiplied by 60. $60 \times 1 = 60$; $60 \times \frac{2}{3} = 40$. The quantities, then, are 60 bushels at 40 cents, and 40 bushels at 60 cents.

SECTION XXXIV.

EQUATION OF PAYMENTS.

If A owes B several sums of money, to be paid at different times, he may desire to pay the whole at once, and consequently to know at what time the whole becomes due. This time is found by making an *equation of the payments, multiplied* by the time, as follows:—

1. A owes B 200 dollars; 100 due Jan. 1, 1844; 100 due Jan. 1, 1846. He wishes to pay it all at once. At what time should he pay it?

Now, on Jan. 1, 1844, A is entitled to the use of 100 dollars for 2 years longer; $100 \times 2 = 200$; equal to the use of 1 dollar for 200 years. If he is to pay the whole together, he must keep the 200 dollars long enough to balance that claim. 200)200(1 year, the answer. The whole should be paid one year from Jan. 1, 1844.

2. A owes B 100 dollars due in 6 months, 200 dollars due in 12 months. In how many months should the whole be paid together?

$$\begin{array}{r} 100\times\ 6=\ 600 \\ 200\times12=2400 \\ \hline 300:\ 300\,)\,3000 \\ \hline \end{array}$$

10 months, — the answer.

The above is the method usually employed, and is sufficiently exact for the necessities of business; but it gives a result a little in favor of the debtor; that is, it makes the equated time a little later than it should be. To find the exact equated time, is a problem far too difficult to be used in ordinary business.

Rule. — Multiply each payment by the length of time before it becomes due. Divide the sum of the products by the sum of all the payments. The quotient will express the length of time in which the whole is due.

3. A owes B several sums, due at different times, as follows: $600 in 2 months, $150 in 3 months, $75 in 6 months. What is the equated time for the whole?

4. A man owes $1000; of which 200 are to be paid in 3 months, 400 in 6 months, and the remainder in 8 months. What is the equated time for the payment of the whole?

5. If I owe $1200, one half to be paid in 3 months, one third in 6 months, and the remainder in 9 months, in what time should the whole be paid?

6. A owes B $640; 150 due in 30 days, 200 due in 60 days, and the remainder in 90 days. What is the equated time for the whole?

7. A merchant buys goods to the amount of $1800; one third to be paid in 30 days, one third in 45 days,

and the remainder in 90 days. What is the equated time for the whole?

8. If I owe $1000, half to be paid in 60 days, and half in 120 days, and if I pay $100 down, what will be the equated time for the remainder?

SECTION XXXV.

SQUARE MEASURE.

[See Section XV., Part I.]

1. There is a field in the form of a square, 15 rods on a side. How many square rods does it contain?

2. If the square be 15½ rods on a side, how many square rods will it contain?

3. How many square rods are there in a square field measuring 17 rods on a side?

4. If the field measure 17½ rods on a side, how many square rods will it contain?

5. What is the contents of a square field measuring 21½ rods on a side?

6. What is the area of a rectangular field, its length being 64 rods, and its breadth 12½ rods?

7. There is a rectangular field, its dimensions being 24½ rods and 76½ rods. What is the area?

8. How many acres are there in a rectangular field, its dimensions being 94 rods and 76½ rods?

9. There is a rectangular field containing 7 acres. Its length is 35 rods. What is its breadth?

10. There is a rectangular farm, its length being 132 rods, its breadth 86½. How many acres does it contain?

11. There is a rectangular lot of land containing 325 acres. It measures on one side 176 rods. What will it measure on the other?

12. There is a board containing 12 square feet. It is 13 inches wide. How long is it?

13. A table contains 15 square feet. It is 4 feet long. How wide is it?

14. A certain room contains 30 square yards. It is 16 feet wide. How long is it?

15. A piece of cloth is $1\frac{3}{4}$ yards wide. How much in length will it require to make 8 square yards?

16. There is a room 15 feet by 18. How many yards of carpeting, $\frac{7}{8}$ of a yard wide, will it require to cover it?

17. How many feet of boards will it require to cover the sides and ends of a barn, as high as to the eaves, (its length is 42 feet, width 34, and height 18,) allowing one fifth of the boards to be wasted in cutting?

18. What will the above-named amount of boards cost at $11.50 a thousand feet?

19. A road, $3\frac{1}{4}$ rods wide, passes through a man's land 1 mile. How much of his land does it take?

20. To what damages will he be entitled, allowing him 28 dollars an acre?

21. There is a right-angled triangle. Its base is 64 rods, and perpendicular 20 rods. How many acres does it contain? (See Sec. XVII., Pt. I.)

22. There is a right-angled triangle. Its base is 84 rods, and perpendicular 26 rods. How many acres does it contain?

23. There is a right-angled triangle. Its base is 49 rods, perpendicular 34 rods. How many acres does it contain?

24. There is a right-angled triangle. Its area is 640 rods; the base is 64 rods. What is the perpendicular?

25. A right-angled triangle has an area of 1092 rods. Its base is 60 rods. What is the perpendicular?

SECTION XXXVI.

DUODECIMALS.

In measuring wood and lumber, the dimensions are taken in feet and inches. As one inch is $\frac{1}{12}$ of a foot, the multiplication of feet and inches by feet and inches is the same as multiplying integers and twelfths by integers and twelfths. Take the following example:—

1. What is the contents of a board 3 feet 7 inches long, and 2 feet 4 inches wide?

$$\left\{\begin{array}{rrrr} & \multicolumn{3}{l}{\text{Operation.}} \\ & 3 & \frac{7}{12} & \\ & 2 & \frac{4}{12} & \\ \hline & 6 & \frac{14}{12}\ \frac{12}{12} & \frac{28}{144}. \\ \hline \textit{Ans.} & 6 & \frac{26}{12} & \frac{28}{144}. \end{array}\right.$$

This answer may be reduced to more simple terms. $\frac{28}{144}=\frac{2}{12}+\frac{4}{144}$; adding $\frac{2}{12}+\frac{26}{12}=\frac{28}{12}$, and this again $=2$ feet $+\frac{4}{12}$; adding the 2 feet to the 6 feet, the answer stands, 8 feet $+\frac{4}{12}+\frac{4}{144}$.

As the fractions decrease in value at a twelve-fold rate, whenever the numerator exceeds 12, the excess may be set down, and the 1 or more carried to the next higher fraction.

2. Multiply 5 feet 2 inches by 11 feet 9 inches.

$$\left\{\begin{array}{rrrr} & 5 & \frac{2}{12} & \\ & 11 & \frac{9}{12} & \\ \hline & 3 & \frac{10}{12} & \frac{6}{144}. \\ & 56 & \frac{10}{12} & \\ \hline \textit{Ans.} & 60 & \frac{8}{12} & \frac{6}{144}. \end{array}\right.$$

To render the operation more simple, call the 12ths or inches, *primes*, (marked ′,) and the 144ths or fractions of the second order, *seconds*, (marked ″;) then begin with the lowest order and multiply, setting each

product in its own place, with the mark appropriate to express its value.

3. Multiply 13 ft. 5 in. by 2 ft. 11 in.

	ft.	′	
	13	5	
	2	11	
	12	3′	7″
	26	10	
Ans.	39	1′	7″

4. Multiply 3 ft. 9 in. by 7 ft. 4 in.
5. Multiply 9 ft. 8 in. by 4 ft. 9 in.
6. Multiply 15 ft. 2 in. by 9 ft. 1 in.
7. Multiply 8 ft. 6 in. by 2 ft. 4 in.
8. What is the contents of a board 14 ft. 5 in. long and 1 ft. 1 in. wide?
9. How many feet in a load of wood 8 ft. 6 in. long, 4 ft. 2 in. wide, and 3 ft. 7 in. high?

Multiply two of the dimensions together, and that product by the third dimension.

10. How much wood in a load 11 ft. 3 in. long, 4 ft. 4 in. wide, 3 ft. 11 in. high?

Divide the cubic feet by 128 for cords, and the remainder by 16 for cord feet, or eighths of a cord.

11. How much wood in a pile 38 ft. 6 in. long, 4 ft. 2 in. wide, and 4 ft. high?
12. How much wood in a load 9 ft. 4 in. long, 4 ft. 3 in. wide, 3 ft. 8 in. high?
13. How much wood in a load 7 ft. 8 in. long, 4 ft. 2 in. wide, 3 ft. 4 in. high?
14. How much wood in a load 8 ft. 2 in. long, 4 ft. wide, 4 ft. 3 in. high?
15. How many cords of wood will a shed contain, whose dimensions inside are 22 ft. 6 in. long, 10 ft. 6 in. wide, 7 ft. 8 in. high?
16. Three men own equal shares in a lot of wood lying in two piles. One pile is 13 ft. 4 in. long, 4 ft. 3 in. wide, 4 ft. 4 in. high. The other pile is 17 ft.

long, 4 ft. wide, 3 ft. 10 in. high. How much wood is each man's share?

See note on page 105.

SECTION XXXVII.

EXTRACTION OF THE SQUARE ROOT.

[See Section XVI., Part I.]

This operation will be best understood by taking first the simplest case, where the number is an exact square, and the root containing only two figures.

What is the square root of 196?

Operation.

	$\dot{1}9\dot{6}$	10, 1st part of the root.
	100	4, 2d part of the root.
20)	96	14, *Ans.*
	80	
	16	
	96	
	00	

Place a period over the unit figure; another over that of hundreds. This will show how many figures there will be in the root; for the square of a number has always either twice as many figures as the number, or one less than twice as many. Find the greatest square of tens in the first period, (in the given example, 100,) and set its root (10) in the quotient. This will be the first part of the root. Square the root. Subtract the square from the first period, and bring down the figures of the next period for a dividend. To the left hand, place double the

part of the root already found for a trial divisor. Find by trial what the next figure of the root must be, and set it down under the first part of the root. This is the second part, or unit figure of the root. [In trying for this figure, remember that it must be so small, that, when the divisor shall be multiplied by it, and the square of itself shall be added to the product, the sum shall not exceed the dividend.] Multiply the divisor by the new figure of the root; to this add the square of the same figure, and subtract the sum from the dividend. If the number is an exact square of two periods, as in the above example, there will be no remainder; and the two parts of the root thus found, when added together, will give the whole root.

2. What is the square root of 225? Of 324?
3. What is the square root of 289? Of 529?
4. What is the square root of 361? Of 729?
5. What is the square root of 625? Of 1024?
6. What is the square root of 784? Of 1296?
7. What is the square root of 841? Of 1849?
8. What is the square root of 961? Of 2601?

If there are more than two periods, first consider only the two left-hand periods, and find their root as above directed; then consider the part of the root expressed by these two figures as the first part with reference to the next figure, (to indicate this, you must annex a cipher,) and work for the next; and so on.

9. What is the square root of 15625?

10. What is the square root of 60516? Of 104976? Of 213444?

Square Root of a Decimal.

If there are decimals in the number, point off each way from the place of units; adding a cipher, if necessary, to make the right-hand period complete.

11. What is the square root of 2.56? Of 12.25?
12. What is the square root of 2.25? Of 20.25?
13. What is the square root of 156.25? Of 132.25?
14. What is the square root of 13.69? Of 21.16?
15. What is the square root of 88.36? Of 53.29?
16. What is the square root of 1.69? Of 1.44?
17. What is the square root of .81? Of .64?
18. What is the square root of .01? Of 6.25?

Square Root of a Vulgar Fraction.

To obtain the square root of a vulgar fraction, find the square root of the numerator, and of the denominator, and write the former over the latter.

19. What is the square root of $\frac{4}{9}$? Of $\frac{16}{25}$?
20. What is the square root of $\frac{9}{16}$? Of $\frac{9}{25}$?
21. What is the square root of $\frac{196}{225}$? Of $\frac{25}{64}$?
22. What is the square root of $\frac{256}{289}$? Of $\frac{625}{961}$?

The correctness of the answer may always be tested by multiplying the answer found, by itself. If correct, it will reproduce the original square.

23. What is the square root of $\frac{1}{9}$? Of $\frac{1}{36}$?
24. What is the square root of $\frac{16}{81}$? Of $\frac{81}{121}$?

Another Method of finding the Root of a Fraction.—Reduce the fraction to a decimal, and proceed as already directed in the case of decimals.

25. What is the square root of $\frac{1}{4}$? The square root of 1 is 1; the square root of 4 is 2. *Ans.* $\frac{1}{2}$. Or reduce $\frac{1}{4}$ to a decimal, =.25; square root, .5, *Ans.*

If the number is not a complete square, annex periods of ciphers, as decimals, and carry the operation as far as desired.

26. What is the square root of 70? Of 80?
27. What is the square root of 90? Of 45?
28. What is the square root of 60? Of 84?
29. What is the square root of 200? Of 120?

30. There is a field in the form of a square, containing 1 acre. How many rods does it measure on a side?

31. There is a right-angled triangle, its hypotenuse measuring 60 rods. What is the sum of the squares of the two legs? (See Sec. XVII., Part I.)

32. There is a right-angled triangle. The squares of its legs added together are 81 rods. What is the length of the hypotenuse?

33. There is a right-angled triangle. Its legs measure, one 25, the other 30 rods. How long is the hypotenuse?

34. Two men start from the same place. One travels 8 miles east; the other, 15 miles north. How far are they then apart?

35. A ladder 40 feet long stands against a house, the foot resting on the ground, on a level with the foundation of the house, and 20 feet distant from it. How far up will it reach?

36. The floor of a room measures 16 feet in length, and 14 feet in width. How long a line will reach diagonally from corner to corner?

37. The two parts of a carpenter's square, one 12, the other 24 inches long, may be regarded as the legs of a right-angled triangle. How long would be the hypotenuse connecting their extremities?

38. There is a room 16 feet long, 14 feet wide, and 10 feet high. How long must a straight line be, reaching from a corner of the room at the bottom to the diagonal corner at the top?

39. There is a room, the length, breadth, and height of which are, each, 10 feet. How far is it from a corner of the room at the bottom to the diagonal corner at the top?

40. There is a room, the length, breadth, and height of which are equal. The distance from a corner at the bottom to the diagonal corner at the top is 18 feet. What is the size of the room?

41. I have a cubic block measuring 4 inches each way. How far apart are its diagonal corners?

42. How large a cube can be cut from a sphere which is 1 foot in diameter?

SECTION XXXVIII.

EXTRACTION OF THE CUBE ROOT.

[See Section XIX., Part I.]

We will first consider those numbers the cube root of which is expressed by a single figure. Every exact cube, of not more than three figures, will have for its root some number less than 10, and, consequently, it will be expressed by a single figure. This root can be found by successive trials.

Examples.

1. What is the cube root of 125?
2. What is the cube root of 216?
3. What is the cube root of 512?
4. What is the cube root of 729?

We will next take perfect cubes, the root of which consists of two figures.

5. What is the cube root of 4096?

Operation.

300	4096	10, 1st part of the root
30	1000	6, 2d part of the root.
330	3096	16, *Ans.*
	1800	
	1080	
	216	
	3096	
	0000	

Rule.—Place a period over the unit figure, and another over that of thousands. Find the greatest cube in the first period, whose root is expressible in tens. Set down this root as a quotient in division. Find the cube of the root, and subtract it from the first period, and bring down the second period as a dividend. At the left hand of this set down three times the square of the root, and under this three times the root; add these together, for a trial divisor. Find, by trial, what the next figure of the root will be, and set it under the first part, already found. Multiply by this figure three times the square of the first part of the root, setting the product under the dividend. Multiply by the square of this figure three times the first part of the root, setting the product underneath the other. Under these set the cube of the root figure last found. Add these three numbers together, and subtract their sum from the dividend. If the work be correct, there will be no remainder. Add together the two parts of the root for the answer.

6. What is the cube root of 2744? Of 205379?
7. What is the cube root of 3375? Of 5832?
8. What is the cube root of 4913? Of 10648?
9. What is the cube root of 9261? Of 15625?
10. What is the cube root of 13824? Of 19683?
11. What is the cube root of 46656? Of 39304?

We will next consider the case where there are more than two figures in the root. The number of figures in the root can always be determined by the number of periods placed over the sum, beginning with units, and placing a period over every third place. If there are more than three periods in the cube, regard, first, only the two left-hand periods, obtaining the first and second figures of the root, just as if they constituted the whole root. Then, after bringing down the figures of another period, add the

two parts of the root, and consider their sum as the first part of the root, and proceed to find the next part. To indicate this, you must annex a cipher to the figures of the root already found.

12. What is the cube root of 1953125?
13. What is the cube root of 2406104?
14. What is the cube root of 3796416?

If there are decimals in the given sum, point off both ways from the units' place, adding ciphers, if necessary, to the decimal, in order to make the period complete.

15. What is the cube root of 15.625?
16. What is the cube root of 35.937?

If the number given is not a perfect cube, add periods of ciphers, and carry out the root in decimals as far as may be desired.

17. What is the cube root of 10?
18. What is the cube root of 20?
19. What is the cube root of 50?
20. What is the cube root of 100?
21. A bushel, even measure, contains 2152 solid inches. What would be the inside measure of a cubic box containing 12 bushels?
22. A gallon, wine measure, contains 231 cubic inches. What must be the inside measure of a cubic cistern containing 10 barrels?
23. What would be the measure of a cubic pile of wood containing one cord?

SECTION XXXIX.

PROPORTION.

[See Section XX., Part I.]

Several changes that may be made in the terms of a proportion are exhibited in page 131. In continuing the subject, we will first state some further changes that may be made in the terms without destroying the proportion.

1. Multiply all the terms by the same number.
2. Divide all the terms by the same number.
3. Add the terms of the first ratio for the first antecedent, and the terms of the second ratio for the second antecedent.
4. Add the terms of the first ratio for the first consequent, and the terms of the second ratio for the second consequent.
5. Instead of the sum of the terms in the third case above, take the difference of the terms.
6. Instead of the sum of the terms in the fourth case above, take the difference of the terms.
7. Raise each term to the same power, as second or third power.
8. Extract of each term the same root.

The result, after each of these operations, will still be a proportion, and may be proved to be so, by multiplying the extremes together, and finding the product, equal to that of the means.

Take the proportion, 4 : 16 : : 9 : 36, and perform on it the first change, using any number you please for a multiplier, and then prove the proportion.

Perform on the same proportion the second change.

Perform the third change.

Perform the fourth change.

Perform the fifth change.

Perform the sixth change.

Perform the seventh change, raising to the second power.

Perform the eighth change, extracting the square root.

Finally, you may, in any case, invert the whole proportion; or, invert the terms of each ratio; or, invert the means or the extremes.

Practical Questions.

1. If 7 lbs. of flour cost 31 cents, what will 196 lbs. cost?

As the smaller quantity is to the larger quantity, so is the price of the smaller quantity to the price of the larger.

2. If 3 cwt. of hay cost 2 dollars, what will 35 cwt. cost?

3. If 4 qts. of molasses cost 38 cents, what will 10 qts. cost?

4. If a horse travels 19 miles in 3 hours, how far will he travel in 11 hours?

5. If the freight of 7 cwt. cost 2 dollars, what will the freight of 20 cwt. cost?

6. If 11 dollars buy 3 cords of wood, how many cords will 50 dollars buy?

7. If 7 bushels of oats last a horse 2 months, how long will 23 bushels last him, at the same rate?

8. A man bought a horse for 84 dollars, and sold him for $93. What did he gain per cent.?

As the whole outlay is to 1 dollar, so is the whole gain to the gain on a dollar.

9. A merchant buys flour at $4.35 a barrel, and sells it for $4.63. What is his gain per cent.?

10. A and B form a partnership in trade. A puts in

$500, and B $300, for the same time. They gain $180. What ought each to share?

As the whole stock is to each one's share, so is the whole gain to each one's gain.

11. C and D trade in company. C puts in 750 dollars, and D $450, for the same time. They gain 240 dollars. How much gain ought each to receive?

12. Two men buy a lot of wood in company for 340 dollars. One takes away 42 cords; the other, the remainder, which was 34 cords. What ought each to pay?

13. Two men hire a sheep-pasture in company for 20 dollars. One keeps 30 sheep in it 14 weeks; the other, 24 sheep, 16 weeks. What ought each to pay?

Find how many weeks' pasturing for a single sheep each one had.

14. Two men purchase a lot of standing grass for $36.50. One takes $3\frac{1}{2}$ tons; the other, $1\frac{3}{4}$ tons. What ought each to pay?

Reduce the quantity of hay to fourths of a ton, and then state the proportion.

15. There is a circular piece of ground, whose diameter is 14 rods. What will be the diameter of a circle containing twice as much?

16. There is a circular piece of ground containing 2.5 acres. What will be the area of a circle, the diameter of which is 3 times as great?

17. There are two similar triangular fields. The smaller contains 3 acres, the larger 4. The base of the smaller is 44 rods. How long is the base of the larger?

18. There are two similar rectangular fields. The smaller is 34 rods wide, and 60 rods long. The other has twice as great an area. What are its dimensions?

19. There is a grindstone 4 feet in diameter. What will be its diameter after half of it is ground off?

20. There are two similar triangular pieces of land. The base of one measures 44 rods. The other piece has an area 7 times as large as the first. What is the length of its base?

21. There are two cisterns of the same shape. One is 5 feet deep. The other has a capacity three times as great. How deep is it?

22. If a ball 5 inches in diameter weighs 14 lbs., what will be the weight of one of the same material 6 inches in diameter?

23. What, on the same supposition, will be the weight of a ball of 7 inches diameter?

24. There are two marble statues of the same form, but differing in size. One is 5 feet high, and weighs 740 lbs. The other is 7 feet high. What will it weigh?

25. If a tree, $2\frac{1}{2}$ feet in diameter at the ground, contains 3 cords of wood, how much will there be in a tree of the same shape, $3\frac{1}{2}$ feet in diameter?

26. There are two similar stacks of hay. The smaller is $11\frac{1}{2}$ feet high, and contains $4\frac{1}{4}$ tons of hay. The larger is 14 feet high. How much hay does it contain, supposing both to be of the same solidity?

27. If an iron field-piece, $5\frac{1}{2}$ feet long, weighs 1140 lbs., how many lbs. will an iron cannon of the same shape weigh, that is $10\frac{2}{3}$ feet long?

28. There are two anchors of similar form. The smaller weighs 1100 lbs. The larger is $2\frac{1}{4}$ times as long. What is its weight?

When a cause and an effect are connected together, the increase of the one is always connected with an increase of the other. If 6 horses eat 20 bushels of oats, we may regard the horses as the cause, and the consumption of the oats the effect; or, if we please, we may regard the oats as the cause, and the support of the horses as the effect. But, in either case, an

increase of one would require an increase of the other. When numbers are connected in this way, in a proportion, having the relation of cause and effect to each other, the proportion is said to be *direct.*

But it often happens that quantities are connected together, not as cause and effect, but as limitations of each other; where an *increase* of one quantity requires a *diminution* of the other.

Thus, if the provisions of a ship's company are sufficient to last 17 weeks, at the rate of 13 oz. of bread per day for each man, it is evident that these quantities, 17 and 13, are not cause and effect, but limitations of each other. If one is increased, the other must be diminished. So, if, with a speed of 11 miles per hour, a journey be performed in 31 hours, it is evident that an increase of one term must diminish the other. When quantities mutually *limiting* each other enter into a proportion, it is called *indirect* proportion. No special rule, however, is needed for the statement of such questions; for you can always determine, by strict attention, whether the statement you make is reasonable.

29. If, with a speed of 11 miles per hour, a journey is performed in 37 hours, how long will it take to perform the same journey with a speed of 15 miles per hour?

30. If a stable-keeper has grain for the supply of 29 horses 43 days, how long will the supply last, if he buys 6 horses more?

31. If a barrel of flour last a family of 7 persons 6 weeks, how long will it last 15 persons?

32. If 42 men can do a job of work in 60 days, how long will it take 53 men to perform the same work?

33. A ship-builder employs 50 men to complete a ship, which they can do in 45 days. If 7 of the men

fail to engage in the work, how long will it take the others to perform it?

34. If 8 yds. of cloth, 7 qrs. wide, cost 54 dollars, what will be the cost of 15 yds. of cloth, of the same quality, 9 qrs. wide?

In this example, the length of the two pieces of cloth will not represent the ratio of their values, for they are of different widths. The answer can be found by two statements. First, regarding the two pieces as of the same width,

$8:15::54:101\frac{1}{4}$ dollars, *first answer.*

Next, taking the width into view,

$7:9::101\frac{1}{4}:130\frac{5}{28}$ dollars, *final answer.*

If we examine the above question, we shall see that in neither of them is the quantity of cloth expressed; but, in the first statement, its length, and in the second, its width. Now, the quantity of cloth is expressed by the length multiplied by the breadth. In the smaller piece, it is $7\times8=56$ qrs. of a square yard; in the larger piece, it is $15\times9=135$ qrs. of a square yard. These numbers, 56 and 135, express the quantities of cloth; and taking these, instead of the dimensions, a single proportion gives us the answer;—

$$56:135::54:130\tfrac{5}{28}.$$

As the question is first stated, you observe that, instead of the numbers which form the ratio, 56 : 135, you have only the *factors* of those numbers given. This is called a *Compound Proportion.*

A compound proportion, then, is one in which two or more of the terms of the simple proportion are expressed in the form of their factors.

Every question containing a compound proportion may be solved by means of two or more simple proportions; or, it may be reduced to one simple propor-

tion, as is seen in example 34, above. This method, however, often requires calculations in large numbers. It may therefore be desirable to have a method given by which the process may be made less tedious.

The following rule is offered, as applicable to all cases of proportion, simple or compound,—direct or inverse. It is short, and the attention required in applying it will afford a good discipline for the reasoning powers.

Rule of Proportion.

Draw a horizontal line. Then examine the conditions of the question, and consider, in the case of each, whether its increase would make the answer greater or smaller. If it would make it greater, set it above the line; if smaller, set it below.

Regard the numbers, thus set down, as the terms of a compound fraction. Cancel common factors. Multiply together the terms that remain, for the answer.

Example.

35. If 8 men build a wall 36 ft. long, 12 ft. high, and 4 ft. thick, in 72 days, when the days are 9 hours long, how many men will build a wall 100 ft. long, 10 ft. high, and 3 ft. thick, in 24 days, when the days are 10 hours long?

Cancelling like factors above and below the line, and multiplying the remaining terms,

Operation.

$$\frac{8\ \ 72\ \ 9\ \ 100\ \ 10\ \ 3}{36\ \ 12\ \ 4\ \ 24\ \ 10} = \frac{300}{8} = 37\tfrac{1}{2} \text{ men, } \textit{Answer.}$$

Explanation.

The question is, "How many men?" "If 8 men will build," &c. Now, if it took 80 men to build the

first wall, instead of 8, it would require more men to build the second; then put 8 above the line. "Build a wall 36 ft. long." If it were 360 feet long, instead of 36, it would take fewer men to build the second wall; therefore put 36 below the line. Pursue the same reasoning with all the other conditions.

36. If 15 horses consume 40 tons of hay in 30 weeks, how many horses will consume 56 tons of hay in 32 weeks?

37. If 1 dollar gain .06 of a dollar interest, in 12 months, how much will 740 dollars gain in 8 months?

38. If a crew of 75 men have provisions for 5 months, allowing each man 30 oz. per day, what must be the allowance per day, to make the provisions last $6\frac{1}{2}$ months?

39. If 18 bricklayers, in 12 days, of 9 hours each, build a wall 175 feet long, 2 feet thick, and 18 feet high, in how many days will 6 men, working 10 hours each day, build a wall 100 feet long, $1\frac{1}{2}$ feet thick, and 16 feet high?

40. If 10 masons lay 160 thousand of bricks in 12 days, working 8 hours per day, how many men will lay 224 thousand in 15 days of 10 hours each?

Partnership.

41. Two men trade in company. One puts in 1000 dollars, the other 2000, for the same length of time. They gain 600 dollars. What is each one's share of the gain?

It is evident that each man's share ought to be in proportion to the sum he put in.

As the whole investment is to each partner's investment, so is the whole gain or loss to each partner's gain or loss.

42. Two men trade in company. One puts in

3500 dollars; the other, 4000. They gain 600 dollars. What is each one's share of the gain?

43. Three men trade in company. The first puts in 3400 dollars; the second, 800; and the third, 1200 dollars. They gain 475 dollars. What is each partner's share?

44. Two men purchase a ship for 11000 dollars. One pays 8000 dollars; the other, 3000. They sell the ship for 9500 dollars. What is each one's loss?

45. Two men trade in company. One puts in 1000 dollars for 6 months; the other puts in 1000 for 18 months. They gain 600 dollars. What is each one's share of the gain?

Here, though the money was equal, it is evident the gain of one ought to be three times as great as the other, because his money was in three times as long.

Where the investments are made for different times, each partner's interest will be expressed by multiplying his money by the time it was in trade. Then, as the sum of all the interests is to each partner's interest, so is the whole gain or loss to each partner's gain or loss.

46. Three men trade in company. The first puts in 400 dollars for 8 months; the second, 1100 dollars for 6 months; the third, 1000 dollars for 7 months. They gain 840 dollars. What is each man's share?

47. Four men trade in company. The first puts in 1200 for 2 years; the second, 1500 dollars for 18 months; the third, 600 for 8 months; the fourth, 900 dollars for $6\frac{1}{2}$ months. They gain 1340 dollars. What is each man's share?

General Rule for Cancelling.

In all operations involving simply multiplication and division, set the multipliers above the line, in the form

of their factors, and the divisors below. Cancel common factors, and the resulting compound fraction will be the answer.

Examples.

48. How many blocks of granite, 16 inches long, 12 inches wide, and 8 inches thick, will it take to make a solid pile 128 feet long, 48 feet wide, and 36 feet high?

Here, the dimensions of the pile constitute the factors, which are to be multiplied together to form the dividend. The dimensions of a block are the factors which are to be multiplied together to form the divisor. The operation, then, is as follows:—

$$\frac{\overbrace{\cancel{12}\times\cancel{4}\times\cancel{32}^{4}}^{\text{Length.}};\ \overbrace{12\times\cancel{4}\times12}^{\text{Width.}};\ \overbrace{12\times3\times12}^{\text{Height.}}}{\cancel{16}\times\cancel{12}\times\cancel{8}}=\overbrace{248832}^{\text{Blocks.}},\ \textit{Ans.}$$

49. How many cubic yards are there in a cellar 60 feet long, 45 feet wide, and 9 feet deep?

50. What are the solid contents, in yards, of a stone wall 42 feet long, 15 feet high, and $4\frac{1}{2}$ feet thick?

As $4\frac{1}{2}=\frac{9}{2}$, the 9 goes with the multipliers, and the 2 with the divisors; thus,

$$\frac{\cancel{6}\times7\times3\times5\times\cancel{9}}{\cancel{3}\times\cancel{3}\times\cancel{3}\times\cancel{2}}=105 \text{ cubic yards, } \textit{Ans.}$$

In all cases, Reduce mixed numbers to improper fractions; and if they are multipliers, write the terms in their natural order, the numerator above, and the denominator below, the line. If they are divisors, invert them.

51. What are the solid contents, in yards, of a wall 48 feet long, $7\frac{1}{2}$ feet high, and $2\frac{1}{4}$ feet wide?

52. What is the cost of digging a cellar 63 feet long, 18 feet wide, and $7\frac{1}{2}$ feet deep, at 20 cents a cubic yard?

As 20 cents $= \frac{1}{5}$ of a dollar, this fraction is a multiplier, giving the answer in dollars. If 20 is used, it will give the answer in cents.

53. How many cords of wood are there in a pile 164 feet long, 4 feet wide, and 12 feet high?

54. How many cords of wood are there in a pile 80 feet long, 32 feet wide, and 7 feet high?

55. What is the value of a pile of wood, at $4.00 a cord, the dimensions of which are 90 feet, 8 feet, and 6 feet?

56. What is the cost of cutting a pile of wood, at 60 cents a cord, the dimensions of which are 44 feet, 8 feet, and $5\frac{1}{4}$ feet?

One eighth of a cord, or a pile, 1 ft. $\times$ 4 ft. $\times$ 4 ft. $=$ 16 feet, is called a *cord foot.*

57. How many cord feet are there in a load of wood 8 feet long, $4\frac{1}{2}$ feet wide, and $3\frac{1}{2}$ feet high?

$$\frac{\cancel{8}\times 9\times 7=63}{\cancel{4}\times 4\times \cancel{2}\times 2=8}=7\tfrac{7}{8} \text{ cord feet, } \textit{Ans.}$$

58. How many cord feet in a load of wood, 8 feet long, $4\frac{1}{2}$ feet high, and $5\frac{1}{4}$ feet wide?

59. How many cord feet in a load of wood, 7 feet 6 inches long, 4 feet 3 inches wide, and 3 feet 4 inches high?

60. What is the value of a load of wood, 8 feet long, $4\frac{1}{2}$ feet wide, and 2 feet high, at $\frac{7}{8}$ of a dollar a cord foot?

SECTION XL.

PROGRESSION.

When a series of numbers is given, each one of which has the same ratio to the number which follows it, the series is called a *progression*.

Progression is arithmetical or geometrical. Arithmetical progression is made by the successive addition or subtraction of a common difference. When the common difference is added to each term, in order to make the succeeding one, the series is called an *ascending* series; as, 1, 3, 5, 7, 9, 11, &c.

When the common difference is subtracted, the series is called a *descending* series; as, 11, 9, 7, 5, 3, 1.

If you know the first term and the common difference of an arithmetical progression, you can write the whole series, for to do this, you have only to add or subtract the common difference for each succeeding term. If the whole series is written out, it is evident you can find by inspection any particular term, as the 7th, the 15th, the 20th, &c. But, if the series be a long one, this may be a very tedious operation.

Suppose the series given above, 1, 3, 5, 7, &c., were continued to 87 terms, and you were required to find what was the last term.

By examining the series, you will see that the 2d term = the 1st + the common difference; the 3d = 1st + twice the common difference; the 4th = 1st + 3 times the common difference; the 5th = 1st + 4 times the common difference; &c. *Any term whatever equals the* 1*st term + the common difference, multiplied by a number one less than that which expresses the place of the term.* The 87th term in the above series, therefore, is $1+2\times86=173$.

1. What is the 38th term of the series 1, 3, 5, 7, &c.?
2. What is the 53d term of the same series? The 91st term? The 89th term? The 107th term?
3. In an arithmetical series, the first term of which is 1, and the common difference 3, what is the 64th term? The 75th term? The 81st term?
4. In the series 1, 5, 9, 13, &c., what is the 40th term? The 67th term? The 80th term?

5. In the series 2, 4, 6, &c., what is the 45th term? What is the 100th term? What is the 200th term?

Hence, if you know the number and place of any term, and the common difference, you may find the first or any other term.

6. If the 5th term of an arithmetical series is 13, and the common difference 3, what is the 1st term? What is the 24th term? What is the 191st term?

7. If the 6th term of a series is 77, and the common difference 15, what is the 2d term? What is the 14th term?

8. If the 22d term in a series is 89, and the common difference 4, what is the 10th term? What is the 43d term?

By knowing the number and the place of any two terms, we may find the common difference.

9. In a certain series, the 4th term is 10, and the 7th term is 19. What is the common difference?

$19-10=9$. Now, this difference, 9, is made by the addition of the common difference three times; for $7-4=3$. The common difference, therefore, is $9\div3=3$.

10. In a certain series, the 5th term is 9, and the 11th term is 21. What is the common difference?

11. In a certain series, the 4th term is 13, and the 9th term is 33. What is the common difference?

If we know the 1st term, the common difference, and the number of terms, we can find the sum of all the terms.

12. How many strokes does a clock strike in 24 hours, from noon to noon?

We might write down the series 1, 2, 3, &c., up to 12, which would express the number of strokes in 12 hours, from noon till midnight; we might write the same series again, for the time from midnight till

noon; and, by adding these numbers together, might obtain the answer. But a much shorter way may be found. To exhibit it, we will write the two series thus:—

1st series,	1	2	3	4	5	6	7	8	9	10	11	12,	from noon till midnight.
2d series,	12	11	10	9	8	7	6	5	4	3	2	1,	from midnight till noon.
	13	13	13	13	13	13	13	13	13	13	13	13	

Sum of both series equal to 12 times 13. But 13 is the sum of the first and last terms, and 12 is the number of terms. Therefore, *The sum of the first and last terms, multiplied by the number of terms, gives the sum of all the terms of both series. Half this number will be the sum of one series.*

13. What is the sum of the series 1, 4, 7, to 20 terms?

First find the 20th term.

14. What is the sum of 50 terms of the series 2, 6, 10?

15. A farmer instructed his boy to carry fencing-posts from a pile to the holes in the ground where they were to be inserted, taking one post at a time. The holes are 12 feet apart, in a straight line, and the pile of posts 30 feet from the first hole. How far must he travel, in carrying to their places 100 posts?

16. If the hours in a whole week were numbered in regular progression, and were struck in this way by the clock, how many strokes would the clock strike for the last hour of the week?

What would be the whole number of strokes in the week?

If we know the first and last terms and the common difference, we can find the number of terms.

17. The first term of a series is 4; the last term is 19; the common difference is 3. What is the number of terms?

The difference between the extremes, 19—4, is 15. This, you know, is the common difference, 3, taken a certain number of times; $15 \div 3 = 5$. There are then 5 additions of the common difference. Now, the number of terms is 1 more than the number of times the common difference has been added. To find the number of terms, then,

Find the difference of the extremes; divide it by the common difference; increase the quotient by 1 *for the number of terms.*

DESCENT OF FALLING BODIES.

18. A body falling through the air falls, in the 1st second, 16.1 feet;* in the 2d second, 48.3 feet; in the 3d second, 80.5 feet. How many feet farther does it fall each second than it fell the second before?

19. Taking the answer to the preceding question as the common difference, and 16.1 as the first term of a series, how far will a body fall in 4 seconds?

20. How far will a body fall in 5 seconds?

21. How far will a body fall in 6 seconds?

22. A stone, in falling to the ground, falls the last second 209.3 feet. How many seconds has it fallen, and from what height?

23. A stone, in falling to the ground, falls the last second 241.5 feet. How many seconds has it fallen, and from what height?

24. If a stone, dropped into a well, strikes the water in 3 seconds, how far is it to the surface of the water?

---

* This is a more exact statement than that made in Part I. (See Olmsted's Natural Philosophy.) It should be remarked, also, that no allowance, in these examples, is made for the resistance of the atmosphere, which always diminishes the speed somewhat, and becomes greater and greater as the speed increases.

SECTION XLI.

GEOMETRICAL PROGRESSION.

A series of numbers, such that each is the same part or the same multiple of the number that follows it, is called a *geometrical series*. The ascending series 1, 3, 9, 27, is of this kind, for each term is one third of that which succeeds it. So, in the descending series 64, 16, 4, 1, each term is 4 times the following term.

The number obtained by dividing any term by the term before it, is called the *ratio of the progression*. Thus, in the first of the above examples, the ratio is 3; in the second example, it is $\frac{1}{4}$.

Let us take the series 2, 6, 18, 54, and observe by what law it is formed. The ratio is 3; the first term, 2. The second term is 2×3, or the first term $\times$ the ratio. The third term is 2×3^2, or the first term $\times$ the second power of the ratio. The fourth term is 2×3^3, or the first term $\times$ the third power of the ratio.

Thus each term consists of the first term multiplied by the ratio raised to a power whose index is one less than the number expressing the place of the term.

1. What is the 7th term in the series 1, 4, 16, &c.?
2. What is the 10th term in the series 3, 6, 12, &c.?
3. A glazier agrees to insert a window of 16 lights for what the last light would come to, allowing 1 cent for the first light, 2 for the second, and so on. What will the window cost?
4. If, in the year 1850, the population of the United States shall be 20000000, and if it shall thenceforward double once in every thirty years, what will be the population in 1970?

To obtain the sum of the terms, when the first and last terms are given, and the ratio,

Take the following question:—What is the sum of a series whose first term is 1, last term 64, and ratio 4?

We can write the series in full, and then obtain the answer by adding the terms together; thus, $1+4+16+64=85$. In this way we might obtain the sum of any given series; but the operation becomes tedious if the series be a long one, and hence a shorter method is devised.

What is the sum of a series whose first term is 2, last term 162, and ratio 3?

To show how a general rule is obtained, we will write this series in full;

$$\text{thus, } 2,\ 2\times3,\ 2\times3^2,\ 2\times3^3,\ 2\times3^4;$$
$$\text{or, } 2,\ 6,\ 18,\ 54,\ 162.$$

If, now, we multiply this series by the ratio, and set the terms of the product one degree to the right, over the terms of the series, we shall have this;

$$\begin{array}{rl} 6+18+54+162+486 & =3\text{ S. or 3 times the sum.} \\ 2+6+18+54+162 \qquad & =\text{S. or once the sum.} \end{array}$$

If, now, we subtract the lower line of terms from the upper, the remainder, it is clear, will be twice the whole sum of the series; for it will be once the sum subtracted from 3 times the sum. But, in this subtraction, all the intermediate terms cancel each other, leaving only the first term, 2, to be subtracted from the last term, 486. Now, this last term, 486, was obtained by multiplying the last term given in the question by the ratio. Therefore we might have saved all our work, and simply multiplied the last term by the ratio, and subtracted the first term from the product. This would have given us, at once, twice the sum of the series, or, in general terms, the series multiplied by a number one less than the ratio. Hence we have the following

RULE.—*Multiply the last term by the ratio, subtract the first term from this product, and divide the remainder by the ratio diminished by one.*

5. A gentleman promises his son, 11 years old, one mill when he shall be 12 years old, and, on each succeeding birthday, till he is 21 years old, ten times as much as on the preceding birthday. What will the son's fortune be, without interest, when he is 21 years old?

6. What is the sum of 14 terms of the series 2, 6, 18, &c.?

First obtain the last term.

7. What is the sum of 16 terms of the series 5, 10, 20, &c.?

8. What is the sum of 18 terms of the series 1, 2, 4, &c.?

9. A builder offers to build a church for 16000 dollars; or, if the people prefer it, he will take for his pay 1 cent for the first pew, 2 cents for the second, 4 for the third, 8 for the fourth, &c., till he shall have received pay for forty pews. What would the meeting-house have cost on these last-named terms?

SECTION XLII.

MENSURATION OF SURFACES.

For the mensuration of the triangle and the parallelogram, when the base and height are known, see Section XVII., Part I.

To find the area of an equilateral triangle, when the sides only are known,

Square one side; multiply that product by the decimal .433.

To find the circumference of a circle, when the diameter is known,

Multiply the diameter by 3.1416.

1. What is the circumference of a circle the diameter of which is 36 feet?

2. What is the circumference of a circular race-course whose diameter is $1\frac{1}{4}$ mile?

3. What is the circumference of a wheel the diameter of which is 24 feet 6 inches?

4. What is the circumference of the earth on the line of the equator, its diameter being 7925.65 miles?

To find the diameter of a circle, when the circumference is known,

Divide the circumference by 3.1416.

5. How far is it across a circular pond, the circumference of which is 231 rods?

To find the area of a circle, when the diameter and circumference are known,

Multiply the circumference by one fourth of the diameter; or, multiply the square of the diameter by the decimal .7854.

6. What is the area of a circle whose diameter is 34 rods?

7. What is the area of a circle whose diameter is 24 feet?

When a circle is given, to find a square which shall have an equal area,

Find the area of the circle, extract the square root, which will be one side of the square.

8. There is a circular piece of land, 40 rods in diameter. What will be the side of a square of equal area?

9. There is a circular green, containing 8 acres. What will be the side of a square of equal area?

10. There is a circle 35 rods in diameter, and a square 31 rods. Which is the greater; and how much?

SECTION XLIII.

MENSURATION OF SOLIDS.

A plane solid is bounded by flat surfaces; a round solid is bounded by curved surfaces.

To find the surface of a solid bounded by plane surfaces,

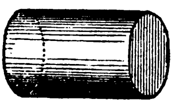
Find the area of each plane surface, and add the sums together for the whole surface.

A prism is a solid, whose ends are any equal, parallel, and similar rectilineal figures, and whose sides are parallelograms.

To find the solidity of a prism,

Multiply the area of the base by the height.

1. What is the solidity of a prism whose ends are equilateral triangles, 14 inches on a side, and whose height is 8 feet?



A cylinder is a round solid, whose ends are equal and parallel circles.

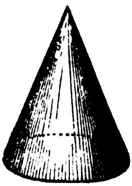
To find the solidity—*the same rule as for a prism.*

2. What is the solid contents of a cylinder whose ends are circles 18 inches in diameter, and whose height is 12 feet?

A regular pyramid is a solid whose sides are equal and similar triangles, meeting in a point at the top. The slant height is the distance from the point, at the top, to the middle of the base of one of the triangles.

To find the solid contents of a pyramid,
Multiply the area of the base by one third the perpendicular height.

3. What is the solid contents of a four-sided pyramid, whose base measures 40 feet on a side, and whose height is 42 feet?



A cone is a round solid, standing on a circular base, and terminating in a point at the top.

To find the solidity of a cone—*the same rule as for a pyramid.*

4. What is the solidity of a cone whose base is 13 feet in diameter, and whose height is 22 feet?

5. What is the solidity of a cone whose base is 4 feet in diameter, and height 18 feet?

To find the surface of a sphere,
Multiply the diameter by the circumference.

6. How many square miles of surface has the earth, regarding it as a sphere the diameter of which is 7925.65 miles?

To find the solidity of a sphere,
Multiply the surface by $\frac{1}{6}$ *of the diameter; or, multiply the cube of the diameter by the decimal* .5236.

To find the measure of a sphere, when the solidity is given,
Divide the solidity by the decimal .5236; *extract the cube root of the quotient, which will be the diameter of the sphere.*

7. What is the solid contents of a sphere the diameter of which is 14 inches?

8. What will be the solidity of the largest sphere that can be cut from a cubic block 1 foot on a side?

SECTION XLIV.

MISCELLANEOUS THEOREMS AND QUESTIONS.

Given the sum and the difference of two numbers, to find the larger and the smaller numbers,

Add half the difference to half the sum, for the larger; subtract half the difference from half the sum, for the smaller.

1. The sum of two numbers is 140, the difference 32. What are the numbers?
2. The sum of two numbers is 572, the difference 94. What are the numbers?
3. The sum of two numbers is 187, the difference 44. What are the numbers?
4. The sum of two numbers is 190, the difference 57. What are the numbers?

Given the sum and the product of two numbers, to find the larger and the smaller number.

5. There are two numbers; their sum is 80, and their product 1551. What are the numbers?

The theorem on which the solution of this question depends, is this:—If a number be divided into two equal parts, and also into two unequal parts, the product of the two equal parts, that is, the square of half the number, will equal the product of the two unequal parts, plus the square of the difference between one of the equal and one of the unequal parts.

Take 16; divide it equally, $8+8$, and unequally, $9+7$; the difference between an equal and an unequal part, 1; $8^2=64:9\times7=63$; loss, 1, which is the square of the difference.

Divide unequally into $10+6$; difference, 2; $8^2=64$:

$10\times 6=60$; loss, 4, which is the square of the difference.

Divide unequally into $11+5$; difference, 3; $8^2=64:11\times 5=55$; loss, 9, which is the square of the difference.

Hence, to solve the question, — *Subtract the product of the unequal parts from the square of half the number; find the square root of the difference; add it to half the number for the greater, subtract it from half the number for the less.*

6. There are two numbers the sum of which is 100. Their product is 2419. What are the numbers?

7. There is a rectangular piece of land, the two contiguous boundaries of which measure together 120 rods. The area of the piece is 2975 square rods. What is its length? What is its width?

8. A rectangular piece of land is surrounded by 480 rods of fence. The area is 13104 square rods. What is its length and breadth?

Given the sum of two numbers, and the difference of their squares, to find the greater and the less number,

The product of the sum and the difference of two numbers is equal to the difference of their squares.

Take the two numbers 6 and 9; their sum is 15; their difference, 3; $15\times 3=45$. The square of 6 is 36; the square of 9 is 81; $81-36=45$.

Hence, if we divide the difference of the squares by the sum, the quotient will be the difference, and from this we may find the greater and the less.

9. There is a triangle, the hypotenuse and one leg of which measure together 90 feet; the other side measures 40 feet. What are the lengths of the two first-named sides respectively?

10. There is a triangle; the hypotenuse and base measure together 120 feet; the perpendicular measures 64 feet. What is the length of each of the first-named sides?

This principle enables you to multiply readily any two numbers, one of which exceeds a certain number of tens by as many units as the other falls short of it; as, 63×57. The first exceeds 60 by 3; the second falls short of it by 3.

Square the tens — 3600; subtract from this the square of the units — 9. 3591, answer.

11. Multiply 64×56. $3600 - 16 = 3584$, answer.

12. Multiply 82×78; 47×33; 92×88.

Theorem of Parallel Sections.

If a line is drawn in a triangle, parallel to one of the sides, and meeting the other two sides, it divides those sides proportionally; and the small triangle cut off is similar to the whole undivided triangle.

If a plane pass through a pyramid or cone, parallel to the base, it divides all the lines it meets proportionally; and the small solid cut off at the top is similar to the whole undivided solid.

13. There is a triangular field, containing 7 acres. A line is drawn through it, parallel to one side, cutting the other two sides $\frac{2}{5}$ of the distance from the apex to the base. How much land does it cut off?

14. There is a board, in the form of a right-angled triangle, 8 feet in perpendicular height. How far from the top must a line be drawn parallel to the base to cut off $\frac{3}{8}$ of the board?

15. A man had a field of 3 acres, in the form of a right-angled triangle, with the base equal to the perpendicular. He sells one acre, to be cut off by a line running parallel to the base. He then sells another

acre, to be cut off by another line parallel to the base. How far from the base must the first line be? How far from the base must the second line be?

16. There was a cone 20 feet high; but, the upper part being defective, 11 feet in height of the top was taken down. How much of the cone has been removed?

17. There was a square pyramid, the base of which measured 48 feet on a side. When it was partly completed, its slant height, measuring from the middle of a side at the bottom to the middle of the same side at the top, was 40 feet, and the width of a side at the top was 12 feet. How high was the apex of the pyramid when completed? And what part of the pyramid remained to be built?

18. In a right-angled triangle, whose base and perpendicular are equal, what is the ratio of the square of the hypotenuse to the square of the base? What is the ratio of the hypotenuse to the base?

19. If a man travel on Monday 6 miles due north, and on Tuesday 8 miles due east, how far is he then from where he set out on Monday?

If, on Wednesday, he travels 12 miles due south, how far will he then be from where he was on Monday morning? How far from where he was on Monday night?

20. In repairing a meeting-house, it was thought desirable to alter the form of the posts, which were one foot square. It was proposed to cut away the corners, so as to make them regular eight-sided prisms. How wide must each face be, so as to have all the eight faces of exactly the same width?

21. Sound moves through the air at the rate of 1090 feet in a second. How long would it be in passing 100 miles?

22. At the above rate, how long would it require for a wave of sound to compass one half the circuit of

the globe, on the line of the equator, the circumference being 24899 miles?

23. Two men purchase, in equal shares, a stick of hewn timber, 40 feet long, 2 feet square at the larger end, and 1 foot square at the smaller end. How far from the larger end shall they cut it in two, so that each may have exactly one half?

24. A surveyor, in laying out a lot of land, first runs a line due north, to a certain tree. From the tree he runs between south and west till he comes to a point due west from the place he started from. The whole of these two lines is 212 rods; but those who measured it neglected to note how far the tree was from the starting point. On measuring a third line, connecting the extremities of the two first lines, they find it 98 rods. How many acres does the triangle contain?

Specific Gravity.

The specific gravity of a body is its weight compared with the weight of an equal bulk of water. To find the specific gravity of a body heavier than water,

Weigh the body in water, and out of water, and find the difference in the weight. Then, as the difference in the weight is to the weight out of water, so is 1 to the specific gravity.

The weight of a cubic foot of water is 62½ lbs. avoirdupois. The specific gravity of the most important of the metals is as follows:—

Iron, 7.78;	Tin, 7.2;	Copper, 8.895;
Lead, 11.325;	Mercury, 13.568;	Silver, 10.51.
Gold, 19.257;	Platinum, 21.25.	

From the above table, we may find the weight of any mass of one of the above metals the magnitude of which is known.

25. What is the weight of a cubic foot of iron?

26. What is the weight of an iron ball six inches in diameter?

27. What is the diameter of a 24 lb. cannon ball?

28. What is the diameter of a 48 lb. cannon ball?

29. What is the weight of a cannon ball one foot in diameter?

30. If the column of mercury in the barometer be $29\frac{1}{2}$ inches high, what would be the weight of a column of mercury of that height one inch square?

31. As the weight of mercury in the barometer equals the weight of the atmosphere on the same base, what is the pressure of the atmosphere on a square foot, when the mercury in the barometer is 29 inches high?

The height to which water will rise in a suction pump, and the height of the mercury in the barometer, are in inverse proportion to the specific weight of those two bodies; that is, the water is as much higher than the mercury, as mercury is heavier than water.

32. How high will water rise in a suction pump, when the mercury in the barometer is $29\frac{1}{2}$ inches high?

33. What is the weight of a copper prism, its base being an equilateral triangle, 3 inches on a side, and its height 15 inches?

Mechanical Powers.

The object to be gained by the application of mechanical powers, is to overcome a large weight or resistance by means of a comparatively small power.

In doing this, however, the *power must move through a space as much larger than the space which the weight moves through, as the weight is heavier than the power;*

Or, the *distance* × *weight* of the *power* = *distance* × *weight* of the *weight or mass to be moved.*

This is the great law of mechanical powers, and applies to them all, without exception. In the practical application of them, a certain allowance must be made on account of the friction in the machine. The amount of friction differs in different powers. No account of this will be taken in the examples which follow, unless it is particularly mentioned; nor will any difference be made between the power when in motion, and when in equilibrium, or at rest.

The Lever.

The lever is a straight bar, used to support or raise heavy weights. It is supported by a *prop* or fulcrum, placed near the weight; and the power is applied at the other end of the lever. The distance from the fulcrum to the weight, is called the *shorter arm*, the distance from the fulcrum to the power, the *longer arm*, of the lever. If the lever were to turn over the fulcrum as a centre, the longer arm would describe a larger circle, and the shorter arm would describe a smaller circle. The circumferences of these two circles, or an arc of the same number of degrees in both, would be the distances passed through by the power and the weight respectively. But we may take the *arms* themselves as representing these distances, for they are the radii of the two circles; and the radii of different circles have the same ratio to each other as the circumferences.

We have, therefore, this proportion;—

The longer arm is to the shorter arm as the weight to the power. Or, let **l. a.** stand for the longer arm; **s. a.,** for the shorter; **w.,** for the weight; and **p.,** for the power.

l. a. : s. a. : : w. : p.; and any change admissible in the terms of a proportion, may be made in these terms.

34. If a lever 10 feet long have its fulcrum one foot from the weight, how great must the power be, to raise a weight of 1640 lbs.?

35. If a lever 10 feet long have its fulcrum 18 inches from the weight, how great a weight will be raised by a power of 160 lbs.?

36. A lever 18 feet long rests on a fulcrum 2 feet from the end. How large a weight can two men raise, (one weighing 164 lbs., the other 172 lbs.,) by applying their weight at the longer arm?

37. If a lever $7\frac{1}{2}$ feet long rest on a fulcrum 15 inches from the end, how heavy must the power be to support a ton gross weight?

38. If the weight be 3600 lbs., and the power 140 lbs., how far from the weight must the fulcrum be placed, under a lever 12 feet long, so as to have the weight and power balance?

39. If the weight be 6480 lbs., the power 312, and the lever $16\frac{1}{2}$ feet long, how far from the weight must the fulcrum be, to have the weight and power balance?

40. In a certain machine, it is necessary to adjust a lever 3 feet long, so that a power of $1\frac{1}{2}$ lbs. shall balance $13\frac{1}{2}$ lbs. How far from the weight must the fulcrum be placed?

The Wheel and Axle.

In this case, the power is applied at the circumference of the wheel, and the weight is drawn up by a rope passing round the axle, which is a smaller wheel. The principle, therefore, is the same as in the lever. The semi-diameter of the wheel is the longer arm; the semi-diameter of the axle, the shorter arm.

41. In a grocery store, the wheel and axle used in raising heavy articles, are of the following dimensions,

viz.: the wheel 5 feet in diameter, the axle 7 inches in diameter. What power must be applied to the rope passing over the wheel, to balance a barrel of flour, weighing 205 lbs., suspended by a rope passing over the axle?

42. With the same wheel and axle, what power will raise a box of sugar, weighing 431 lbs., adding $\frac{1}{5}$ to the power, to overcome the friction?

43. In digging a well, the wheel employed in raising stones and earth, is 6 feet in diameter; the axle, $6\frac{1}{2}$ inches in diameter. What power will raise a rock weighing 640 lbs., adding $\frac{1}{4}$ to the power, to overcome the friction?

44. If a wheel is 14 feet in diameter, what must be the diameter of the axle, in order that a power of 140 lbs. may balance 5760 lbs.?

45. If an axle is $16\frac{1}{2}$ inches in diameter, what must be the diameter of the wheel, in order that a power of 56 lbs. may balance a weight of 1344 lbs.?

The Screw.

In this case, the distance passed through by the power in one revolution, is equal to the circumference of the circle described by the lever which turns the screw. The distance passed by the weight is the distance between two threads of the screw, measured in the direction of its axis.

In the practical application of this power, a large allowance must be made to compensate for the friction.

46. If the lever of a screw is 11 feet in length, and the distance of the threads $1\frac{1}{4}$ inches, what power will raise a weight of 6431 lbs., making no allowance for friction?

47. With the same conditions as in the last example, what weight will be raised by a power of 124 lbs.?

48. What must be the length of the lever of a screw, the threads of which are 1 inch asunder, in order that a power of 3 lbs. may balance a weight of 1640 lbs., making no allowance for friction?

49. How far asunder must the threads of a screw be, so that, with a lever of 8½ feet in length, 26 lbs. will balance 6590 lbs.

Strength of Beams to resist Fracture.

[See Section XX., Part I.]

In addition to the principles that have already been stated in estimating the strength of timbers, the following are among the most important. It is understood, in all cases, when timbers are compared, that they are of the same wood, and equally good in quality.

When the depth of two beams is the same, and the thickness the same, the *strength is inversely as the length.*

50. There are two beams, of the same depth and thickness; one, 18 feet in length; the other, 13. The longer beam will sustain a weight of 68 cwt. What weight will the shorter beam sustain?

51. Two beams, of the same size, measure in length 22 and 17½ feet. The shorter beam will sustain 76 cwt. How much will the longer beam sustain?

52. Two beams, of equal thickness, have a depth of 14 and 16 inches respectively. The deeper beam is 20 feet long, and will sustain 84 cwt. The other is 17 feet in length. What weight will it sustain?

First take into view the length; then, in a second proportion, the depth.

53. If a beam, 25 feet in length and 9 inches in depth, will sustain a weight of 12 cwt., what weight

will be sustained by a beam of the same thickness 18 feet long, and 10 inches in depth?

When beams are of the same length and depth, *the strength varies directly as the width.*

54. There are two beams of equal length and depth; one, 9 inches in width; the other, $7\frac{1}{2}$ inches. The wider beam will sustain 47 cwt. What weight will the narrower beam sustain?

55. There are two beams of equal depth. One measures 20 feet in length, and 11 inches in width, and will sustain 94 cwt. The other beam is 14 feet in length, and 10 inches in width. What weight will it sustain?

56. There are two beams, of the same width. One measures 16 feet in length, and 10 inches in depth, and will sustain 66 cwt. The other is 18 feet long, and 12 inches in depth. What weight will it sustain?

It is sometimes desirable to know how the strength of a beam will vary by removing the point on which the pressure is made, as in the following example:—

57. A beam 20 feet in length will sustain, at its centre, a weight of 44 cwt. What weight will it sustain applied 7 feet from one end?

The following formula will give the variation in the strength:—

As the product of the two unequal sections of the beam (in this case, 13×7) is to the square of half the length, so is the weight which the beam will sustain at the centre to the weight it will sustain at the other given point.

58. A beam 24 feet in length will sustain at its centre 56 cwt. What weight will it sustain at the distance of 9 feet from one end?

59. A beam 28 feet in length will sustain at its centre 33 cwt. What weight will a beam of the same width and length, and of $\frac{3}{4}$ the depth of the former, sustain at the distance of 10 feet from the end?

Stiffness of Beams to resist Flexure.

The stiffness of beams of the same length and width varies as the *cube of the depth.* If the depth is the same, the *stiffness varies as the width.*

60. There are two beams, of equal length and width. One is 8 inches in depth; the other, 11 inches. If it require 30 cwt. to bend the former 1 inch, what weight will it require to bend the latter 1 inch?

61. There is a stick of timber 8 inches by 6. If it require 24 cwt. to bend it 2 inches when lying flat, what weight will bend it 2 inches when turned up on the edge?

62. If 10 cwt. will bend the stick just described $1\frac{1}{2}$ inches when it lies flat, what weight will be requisite to bend it $1\frac{1}{2}$ inches when turned up on the edge?

63. There is a board 12 inches wide, and 1 inch in thickness. What is the ratio of its strength when lying flat, supported at the ends, to its strength when turned edgewise?

64. If it require 12 lbs. to bend the same board $\frac{1}{2}$ an inch, when lying flat, how much will it require to bend it $\frac{1}{2}$ an inch when turned edgewise?

SECTION XLV.

BUSINESS FORMS AND INSTRUMENTS.

Promissory Notes.

1.—*On Demand, with Interest.*

$500.—Boston, *March* 1, 1846. For value received, I promise A. B. to pay him, or his order, five hundred dollars, on demand, with interest. T. M.

2.—*On Time, with Interest.*

$200.—Boston, *March* 1, 1846. For value received, I promise A. B. to pay him, or his order, two hundred dollars, in three months, with interest.
T. M.

3.—*On Time, without Interest.*

$400.—Boston, *March* 1, 1846. For value received, I promise A. B. to pay him, or his order, four hundred dollars, in sixty days from date. I. M.

4.—*Payable by Instalments, with Periodical Interest.*

$1000.—Boston, *March* 1, 1846. For value received, I promise A. B. to pay him, or his order, one thousand dollars, as follows, viz.;—two hundred dollars in one year, two hundred dollars in two years, and six hundred dollars in three years, from this date, with interest semi-annually. I. M.

Remarks on Promissory Notes.

When the words "or order" are inserted in a note, the holder of the note may endorse it, that is, write

his name on the back of it, and pass it to a third person, who can collect it in the same manner as if he were the original holder. If the maker of the note neglects to pay, the holder may collect it of the endorser.

If the words "or bearer" are inserted instead of "or order," any person who has possession of the note may collect it of the maker. Such a note would be like a bank note, which passes from hand to hand without endorsement.

A note, in order to be legal in the first holder's hands, must be for value received. A note, therefore, given to pay a debt incurred in gambling or betting, cannot be collected by law, unless it has passed into the hands of an innocent holder.

When a note contains the promise to pay interest annually, and the interest is not collected annually, the law does not permit the holder to draw compound interest. The holder may compel the payment of the interest when it becomes due; but if he neglect to do this, he can recover only simple interest.

When a note is given to pay in a certain commodity, as wood, grain, &c., if the note is not paid when due, the holder may compel the payment of the equitable value of the commodity in money. The reason of this is, that it is supposed that the commodity may have a value to the holder at the time when it is promised, which it will lose if not paid then.

Receipts.

1. — *A general Form.*

$500. — Boston, *March* 1, 1846. Received of O. P. the sum of five hundred dollars, in full of all demands against him. A. B.

2.—*For Money paid by another Person.*

$300.—Boston, *March* 1, 1846. Received of O. P., by the hand of Y. Z., three hundred dollars, in full payment for a chaise by me sold and delivered to the said O. P. A. B.

3.—*For Money received for Another.*

$700.—Boston, *March* 1, 1846. Received of O. P. seven hundred dollars, it being for the balance of account due from said O. P. to Y. Z. A. B.

4.—*In Part of a Bond.*

$3000.—Boston, *March* 1, 1846. Received of O. P. the sum of three thousand dollars, being a part of the sum of five thousand dollars due from said O. P. to me on the ——— day of ———. A. B.

5.—*For Interest due on a Bond.*

$600.—Boston, *March* 1, 1846. Received of O. P. six hundred dollars, due this day from him to me, as the annual interest on a bond, given by said O. P. to me on the 1st of May, 1831, for the payment to me of ten thousand dollars in three years, with interest annually. A. B.

6.—*On Account.*

$50.—Boston, *March* 1, 1846. Received of O. P. fifty dollars, for which I promise to account to him on a settlement between us. A. B.

7.—*Of Papers.*

Boston, *March* 1, 1846. Received of O. P. several contracts and papers, which are described as follows:—[*describe the papers;*] which I promise to return to the said O. P. on demand. A. B.

ORDER AT SIGHT.

BOSTON, *April* 18, 1846. At sight, pay to the order of John Brown, one thousand dollars, value received, which place to account of

Your obedient servants, A. W. & Co.

JACOB SMITH, Esq., *New York*.

ORDER ON TIME.

BOSTON, *April* 18, 1846. Six months after date, pay to the order of John Brown, one thousand dollars, value received, which place to the account of

Your obedient servants, A. W. & Co.

JACOB SMITH, Esq., *New York*.

Remarks. — If J. B. present this order to J. S., and J. S. write his name across the face of it, it becomes what is called an *acceptance*. J. S. agrees to pay it at the date named.

If J. B. writes his name on the back of the acceptance, it becomes negotiable. He may pass it to a third person, who may endorse it, and pass it to a fourth. All those whose signatures are on the order are bound for its payment; — the acceptor to the drawer; the acceptor and drawer to the first endorser; and they and each endorser to the one succeeding him; and the last endorser, and all previous parties, to the holder.

AWARD BY REFEREES.

We, the undersigned, appointed by agreement of the parties herein named, having met the parties, and heard their several allegations, arguments, and proofs, and duly considered the same, do award and determine that A. B. shall recover of C. D. the sum of ———, together with all the costs of this reference, which are to the amount of ———; and that this shall be final and in full of all claims and dues of the parties on matters herein referred to us.

I. M.
R. N.
L. S.

Letter of Credit for Goods.

Boston, *March* 1, 1846.

Messrs. Y. & Z., Merchants, *Baltimore.*

Gentlemen,—Please to deliver Mr. C. D., of ——, or to his order, goods and merchandise to an amount not exceeding in value, in the whole, one hundred dollars; and, on your so doing, I hereby hold myself accountable to you for the payment of the same, in case Mr. C. D. should not be able so to do, or should make default, of which default you are required to give me reasonable and proper notice.

Your obedient servant, A. B.

A letter of credit for money may be given in the general form of the above; specifying, in the letter, the amount of credit granted.

Power of Attorney.

Know all men by these presents, that I, A. B., of ———, do hereby appoint C. D., of ———, to be my sufficient and lawful attorney, to act for me, and in my name, [*here state the objects for which he is to act.*] And for the purposes aforesaid, I hereby grant unto my said attorney full power to execute all needed legal instruments, to institute and prosecute all claims in my behalf, to defend all suits against me, to submit to arbitration, or settle all matters in dispute, and to do all such acts as he shall think expedient for the full accomplishment of the objects for which he is appointed my attorney, as fully as I might myself do them if present; and all acts done by the said C. D., my attorney, under authority of this appointment, I will ratify and confirm.

In testimony whereof, I hereby set my hand and seal, this ——— day of ———, in the year ———.

A. B. [L. S.]

Signed, sealed, and delivered,
in the presence of S. N.
W. F.

SECTION XLVI.

ON THE STANDARD OF WEIGHTS AND MEASURES.

In the earlier states of society, the standard of weights and measures was, of necessity, very indefinite and fluctuating. In one nation it was one thing; in another nation, another; and in no case was it deserving of a very high degree of confidence.

Sometimes the length of the king's foot was the standard for all measures of length; again, the length of the king's arm, from the elbow to the extremity of the fingers, was made the standard.

The length of journeys was measured by the hours or days employed in performing them, or by the number of steps taken.

In land measure, the standard was, what a yoke of oxen could plough in a day, when, in fact, one yoke might plough twice as much as another.

In dry measure, it was as much as a man could conveniently carry, without first deciding how strong the man should be.

In weight, the standard was, what a man could hold and swing in his hand.

Sometimes vegetables were taken as measures; as, "three barley corns make one inch." But barley corns do not all grow of exactly equal length, any more than the feet and arms of kings.

As science advanced, and commerce became farther extended among different nations, the mischiefs of these vague and fluctuating methods of measurement became more and more deeply felt.

But it was far easier to see the faults of the old system, than to devise a new one that should be perfect. What object could be selected as an ultimate standard for all weights and measures?

We have seen that the parts of animals or of vegetables are too liable to change to deserve any confidence. If some arbitrary standard should be adopted, as a foot, or yard, and this measure should be kept as the standard, by which all others should be tried, what security could there be that it would never be altered by fraud or destroyed by accident? Or, if some natural distance were taken, as the distance between two points of some well-known rock or cliff, this distance might vary with a change of temperature or be altered by some convulsion of nature.

We will proceed to give a short account of the English system of weights and measures, adopted by their *Act of Uniformity*, which took effect Jan. 1, 1826. To begin with measures of capacity; all English measures of capacity, whether for liquors or grain, are referred to the *standard imperial gallon.* This gallon contains 277¼ cubic inches. From this gallon, quarts, pints, and gills, are obtained by subdivision; and pecks and bushels by multiplication. Hence you can find the number of cubic inches in an English quart, pint, peck, or bushel. Thus the adoption of the imperial gallon introduces entire uniformity into all English measures of capacity. It refers them all ultimately to the cubic inch. We must now inquire, what has been done to fix the measure of the inch; for, if there is any error or variation here, it will render false all the measures of capacity which depend upon it.

To determine the measure of the inch, it is made by law $\frac{1}{36}$ of the *standard yard.* That standard yard is a straight brass rod in the custody of the clerk of the House of Commons. The yard is the distance on that rod between the centres of the points in the two gold studs or pins in the rod. And as heat would make the yard longer, and cold would make it shorter, the law requires that it shall be used when it is of the

temperature of 62° (Fahrenheit.) This *standard yard*, however, may be destroyed by accident. We must then inquire for a still more permanent standard. To effect this, the law declares that the standard yard, if destroyed, may be restored, by making it $\frac{360000}{391393}$ of the length of a pendulum, that vibrates seconds in the latitude of London, in a vacuum, at the level of the sea. If all these conditions are fulfilled, a pendulum that vibrates seconds must have an absolutely invariable length.

Thus we have brought the whole system of measures back to seconds, as the standard. The whole scheme now depends upon seconds being of an invariable length.

Seconds are parts of a year. The year is not made up by multiplying seconds, but seconds are obtained by dividing the year. If, then, the year is of a fixed length, seconds are so. Now, the year is the time of the revolution of the earth round the sun. It is the same, without change, from one year to another, and from century to century.

Thus the whole system of measures has been brought, for its ultimate standard, to the unalterable period of the earth's revolution round the sun.

We will now retrace the steps of this investigation, beginning with the primary standard, the earth's yearly revolution.

The time of the earth's revolution round the sun is always the same. Therefore, a second, which is a certain part of this time, is an exact measure. If the second is a fixed measure, then the pendulum which, under the same circumstances, vibrates seconds, is of a fixed length. If the length of the pendulum vibrating seconds is fixed, the length of the standard yard is fixed, for it is $\frac{360000}{391393}$ of the pendulum. If the standard yard is fixed, the inch is fixed; consequently the cubic inch, the gallon, quart, pint, gill, and bushel.

In the preceding investigation, no mention has been made of the standard of weight. It is obtained by making a cubic inch of distilled water equal to 252.458 grs., of which 5760 make a pound Troy, and 7000 make a pound avoirdupois.

Thus weights and measures are alike brought to an unalterable standard.

The imperial gallon contains	277¼	cubic	inches.
The Winchester* gal., wine measure,	231	"	"
" " " beer measure,	282	"	"

The imperial *gallon of water weighs* 10 *lbs.* avoirdupois.

The system of weights and measures established by law in the United States, is very nearly the same as the English. The gallon, United States measure, contains 9 lbs. 14 oz. of water. This is the *legal* standard for all measures, dry and liquid. In many parts of the country, however, especially in the interior, the legal standard has not supplanted the system derived in earlier times from the English.

In France, where the system of weights has been carried to greater perfection than in any other country, the decimal ratio is adopted in all denominations. In some cases, however, there is still retained some part of the old system, combined with the decimal.

In obtaining an ultimate standard of measure, the French measured one quarter of a meridian line of longitude.

One ten millionth part of this arc they made the basis of their system of measures. This standard, the *metre*, is 3.28 feet. The lower denominations are made by successive divisions of this, by 10, 100, &c., and the higher by multiplication.

---

* Winchester; so called because the standard measures were kept at Winchester.

The following table presents the French decimal weights and measures, with the English equivalents.

French Long Measure.

		feet.
10 millimetres make	1 centimetre, ...	.0328
10 centimetres, ...	1 decimetre, ...	.328
10 decimetres, ...	1 metre, ...	3.28
10 metres, ...	1 decametre, ...	32.8
10 decametres, ...	1 hectometre, ...	328
10 hectometres, ...	1 kilometre, ...	3280
10 kilometres, ...	1 myreametre, ...	32800

French Square Measure.

The unit square measure is the *are*, which is the square of the *decametre;* consequently, it is the square of 32.8 feet,—a little less than 4 square rods.

This unit is multiplied for the higher denominations, and divided for the lower, in the same way as the *metre*.

French Decimal Weight.

		grs. Troy.
10 milligrammes make .	1 centigramme, . . .	.1543402
10 centigrammes, ...	1 decigramme, . . .	1.543402
10 decigrammes, ...	1 gramme, ...	15.43402
10 grammes, ...	1 decagramme, . .	154.3402
10 decagrammes, ...	1 hectogramme, . .	1543.402
10 hectogrammes, ...	1 kilogramme, . . .	15434.02
10 kilogrammes, ...	1 myriagramme, . .	154340.2
10 myriagrammes, ...	1 quintal, ...	1543402.
10 quintals, ...	1 million, ...	15434020.

MISCELLANEOUS EXAMPLES.

The examples that follow are designed to carry still farther the practice in Written Arithmetic.

1. James Ball bought of Amos Sewall three pieces broadcloth, measuring $12\frac{1}{2}$, 13, and 24 yards, at \$$4.87\frac{1}{2}$ per yard; five pieces kerseymere, measuring $24\frac{1}{2}$, 25, 27, $26\frac{1}{4}$, and 26 yards, at 67 cts. per yard; eight pieces cotton sheeting, measuring 33 yards each, at $9\frac{1}{4}$ cents per yard; $\frac{1}{2}$ per cent. off. What was the amount of the bill?

2. Bought of John Jones, on six months' credit, 47 yards broadcloth, at \$4.31 per yard; 16 yards vestings, at \$1.15 per yard; $63\frac{1}{2}$ yards satinet, at $62\frac{1}{2}$ cents per yard; $5\frac{1}{2}$ pieces sheeting, containing 33 yards a piece, at $8\frac{3}{4}$ cents per yard. John Jones agrees, if paid in cash, to deduct 4 per cent. from the bill. What is the cash amount of the bill?

3. Bought of Asa Wood, on six months' credit, 45 barrels flour, at \$5.37 per barrel; four hhds. molasses, containing $124\frac{1}{2}$, 131, 134, and 136 gallons, at $27\frac{1}{2}$ cents per gallon; five bags coffee, containing $54\frac{1}{2}$, 56, $49\frac{1}{2}$, 62, and $65\frac{1}{2}$ lbs., at $8\frac{1}{4}$ cents per pound. Gave, in payment of the above, a note payable in six months. What was the cash value of the note when it was given, reckoning interest at 6 per cent.?

4. A bought of B, on six months' credit, goods with the amount as follows: —

April 3, 1845,	\$254.75.
June 8, "	135.00.
Aug. 1, "	200.00.
Sept. 14, "	168.25.

At what date shall A make an equated payment of the whole amount?

5. A gives B a note for \$600, payable in six months. What is the cash value of the note, two months after date, reckoning the interest at 6 per cent.?

6. A man agrees to dig and stone a well on the following terms:—\$1.00 per foot for the first 10 feet, \$2.50 per foot for the second 10 feet, and \$4.00 per foot for the remainder, till he finds a supply of water. For every foot of rock through which he digs, he shall receive double pay. He digs 42 feet in all, and through rock from 17 to $31\frac{1}{2}$ feet from the surface. What pay is he entitled to for the whole?

7. A man engages to build 160 rods of road, one half for \$1.42 per rod, the other half for \$1.83 per rod. He hires 93 days of men's labor at 84 cents per day; pays for board of the same at \$1.50 per week of six days; pays for tools and repairs, \$11.60. He works himself, with 4 oxen and his son, 34 days. What wages will he receive per day for himself, for his oxen, and for his son, allowing for the 4 oxen as much as for himself, and for his son half as much?

8. A bought a lot of standing wood for 105 dollars, and agreed with B to cut and haul it to market for three fifths of the proceeds. There were 54 cords of pine, which was sold for \$2.84 per cord, and 61 cords of hard wood, which sold for \$4.75 per cord. Did A gain, or lose, and how much? B labored himself 35 days, employed one yoke of oxen 24 days, and hired 68 days of men's labor at 85 cents per day. What pay does he receive per day for himself and for his oxen, allowing for his oxen, per day, two thirds as much as for himself?

9. A and B bought a quantity of grass, ready for cutting, for 26 dollars, for which they paid 13 dollars apiece. In cutting and curing it, A furnished 3 days of hired men's labor, and worked himself $2\frac{1}{2}$ days; B hired 7 days of men's labor, a team $1\frac{1}{2}$ days, and worked himself $2\frac{1}{2}$ days. There were $6\frac{1}{2}$ tons of hay.

What is each one's share of the hay, allowing for the labor $1.00 per day, and for the whole work of the team $1.50?

10. A agrees to dig and stone a cellar 7 feet deep. It is to be 13 feet wide and 16 feet long inside the walls, which are to be 2 feet in thickness. He is to receive for digging 22 cents a cubic yard for earth, and $1.55 a cubic yard for rock, and for stoning 87 cents for every perch of 25 cubic feet for which the stone is found in digging the cellar, and $1.50 a perch when he has to bring the stone from another place. In digging the cellar, he digs 7½ cubic yards of rock, which furnishes stone for 7½ cubic yards of wall. What is he to receive for the whole job?

11. A man agrees to dig a canal 10 rods long, 6 feet in depth, 33 feet wide at the top, and 24 feet wide at the bottom, for 10 cents per cubic yard. What sum will the work amount to?

12. What is the value of six loads of wood, at $4.67 per cord, measuring as follows: — first, 8 ft., 4 ft. 3 in., 3 ft. 10 in.; second, 8 ft. 4 in., 4 ft. 1 in., 4 ft.; third, 8 ft. 2 in., 4 ft. 7 in., 3 ft. 11 in.; fourth, 8 ft., 4 ft. 2 in., 4 ft.; fifth, 8 ft. 2 in., 3 ft. 10 in., 3 ft. 9 in.; sixth, 8 ft. 6 in., 4 ft., 4 ft. 4 in.?

13. What will be the dimension, at the two ends, of the largest square stick of timber that can be hewn from a round log 3 feet in diameter at the larger end and 2 feet in diameter at the smaller?

14. What will be the solid contents of such a stick, if it is 16 feet in length?

15. What will be the width at the two ends of the largest stick of timber that can be hewn from a log 3 feet in diameter at the larger end, and 2 feet in diameter at the smaller, if the stick is hewn 10 inches in thickness through its whole length?

16. What will be the solid contents of such a stick, if it is 20 feet in length?

17. There are three pieces of cloth. The length of the first is to that of the second as 3 to 2, the length of the second is to that of the third as 4 to 15, and the length of the three added together is 50 yards. What is the length of each piece?

18. A man bought a chaise, and paid 20 dollars for repairing it. He then sold it for one fifth more than he gave, and found that, allowing one dollar for his own trouble in the business, he had lost thirteen dollars. What did he give for the chaise?

19. A gentleman began his preparation for college at a certain age, and spent in school and at college half as many years as he had lived before. He then went to Europe, and, after spending there one ninth as many years as his age amounted to when he left Europe, spent in his profession one third as many years as he had lived when he entered it, and was then 36 years old. At what age did he begin his preparation for college?

20. One half of three fourths of A's age equals one sixth of B's age, and the sum of their ages is 78. What is the age of each?

21. Three fourths of the liquor in a cask equals five sixths of what has leaked out; and the whole, before any leaked out, was sixty gallons. How much is there in the cask?

22. Reduce 5 pence to the decimal of a shilling, carrying the decimal to the sixth figure.

23. Reduce $9\frac{1}{2}$ gallons to the decimal of a barrel, wine measure, carrying the decimal to the tenth figure.

24. Reduce $7\frac{1}{3}$ quarts to the decimal of a bushel.

25. What is the least common multiple of 784, 1386, and 1235?

26. What are the prime numbers between 1010 and 1020?

27. What are the prime factors of 2326?

28. Reduce $\frac{17}{22\frac{1}{3}}$, $\frac{15\frac{1}{7}}{19}$, and $\frac{21}{30\frac{1}{4}}$, to simple fractions, with a common denominator.

29. What is the value, in shillings and pence, of the following decimals, when added together: — £.0431; .67142s.; .73462d.?

30. What is the interest of $642.25 for 1 year and 3 months, at $7\frac{1}{2}$ per cent.?

31. What is the interest of $954.30 for 2 years, 4 months, and 21 days, at 5 per cent.?

32. What is the present worth of a note of $640.50 due in 3 months?

33. What is the present worth of a note of $1263.00 due in $3\frac{1}{2}$ months, at 5 per cent. interest?

34. BOSTON, *June* 14, 1836. For value received, I promise to pay John Ball, or order, three hundred and sixty-five dollars in four years with interest annually.

JAMES FROST.

If no interest is paid, and the note is renewed annually, what will be the amount of the note four years after date?

35. NEW YORK, *July* 1, 1840. For value received, I promise to pay Abel Jones, or order, five hundred and forty dollars, on demand, with interest.

JOHN FROST.

Endorsements, —	Oct. 3, 1840, ...	$63.00
	Feb. 4, 1841, ...	120.00
	June 1, 1841, ...	60.00
	Sept. 15, 1841, ...	200.00

What will be due Feb. 14, 1842, at seven per cent. interest?

36. A owes B $400.00, due in 2 months; $320.00, due in 3 months; $600.00, due in $4\frac{1}{2}$ months. What is the equated time for the payment of the whole?

37. What is the present worth of a bank note for 800 dollars payable in three months?

38. What is the present worth of a bank note for 346 dollars payable in six months?

39. For what sum must I give a note to a bank payable in three months, in order to obtain 674 dollars?

40. Bought seven 100 dollar shares of bank stock at $6\frac{1}{4}$ per cent. advance. Gave in payment nine 60 dollar shares of railroad stock at 3 per cent. discount, and a bank note payable in 60 days. What was the face of the note?

41. How many square inches are there in $15\frac{1}{2}$ square rods?

42. What is the cost of plastering the sides, ends, and ceiling of a room $22\frac{1}{2}$ feet long, $17\frac{3}{4}$ feet wide, and 11 feet 1 inch in height, at 18 cents per square yard, making no deduction for windows, doors, or woodwork?

43. There is a house 40 feet in length, and 26 feet in breadth. From the beam to the ridgepole is 11 feet. The roof projects 7 inches beyond the walls at each end, and the line of the eaves is 8 inches, measured horizontally, from the side walls. How many square feet are there in the roof?

44. A painter agrees to paint the outside of a house which is 44 feet long, 28 feet wide, $22\frac{1}{2}$ feet in height to the top of the beam, and 11 feet 8 inches from the beam to the ridgepole, for 44 cents per square yard. What is he entitled to for the job, making no deduction for windows, and no addition for cornices or other projections?

45. What is the square root of 9743 to three places of decimals?

46. What is the square root of 17431 to two places of decimals?

47. The base of a right-angled triangle is 744 feet, the hypotenuse 834 feet. What is the perpendicular, to two places of decimals?

48. The base of a triangle is 76 rods, the sum of the hypotenuse and perpendicular is 186 rods. What is the length of the hypotenuse?

49. From a cylinder 12 inches in diameter, it is desired to cut the largest possible four-sided prism, whose opposite sides shall be parallel, and whose width shall be to its thickness as 2 to 1. What will be its width, and what its thickness?

50. What are the dimensions of the largest prism, with parallel sides, that can be cut from a cylinder 12 inches in diameter, making the width to the thickness as 3 to 1?

51. What are the dimensions of the largest prism, with parallel sides, that can be cut from a sphere 15 inches in diameter, making the length and breadth equal, and each of them double of the thickness?

52. What is the cube root of 674 to three places of decimals?

53. What is the cube root of 1736 to two places of decimals?

54. What is the cube root of 31 to three places of decimals?

55. Two men purchase a lot of land for 750 dollars. One pays \$406.50; the other, the remainder. They expend \$341 in equal shares on its improvement, and sell the land for \$1430.00. What is each one's share of the gain?

56. 1841 are how many times four-fifths of $76\frac{1}{2}$?

57. How many bottles, each containing $1\frac{3}{8}$ pints, can be filled from a hogshead containing 63 gallons, allowing a loss of one eleventh in the process?

58. What is the value, in Federal money, of £456 at $9\frac{1}{2}$ per cent. advance?

59. How many gallons, each containing 231 cubic inches, will fill a cylindrical cistern 4 feet in diameter and 5 feet deep?

60. There is a cylindrical cistern 6 feet deep, con-

taining 10 barrels of $31\frac{1}{2}$ gallons each, each gallon containing 231 cubic inches. What is the diameter of the cistern?

61. There is a cistern, in the form of an inverted cone, 8 feet deep, and of the same capacity as the cistern last named. What is its diameter at the top?

62. Bought 74 barrels of flour at \$4.56 per barrel, and, after keeping it 35 days, sold it at \$5.16 per barrel. What per cent. did I gain, allowing 6 per cent. interest on the money invested?

63. The first and fourteenth terms of an arithmetical series are 3 and 19. What is the common increase?

64. The fifth term of an arithmetical series is 18, the 16th term is 39. What is the first term?

65. What is the sum of an arithmetical series, the extremes of which are 9 and 164, and the number of terms forty?

66. Find the ninth term of a geometrical series whose first term is 2 and ratio $\frac{3}{4}$.

67. What is the fifth term of a geometrical series whose second term is 4 and ratio $\frac{2}{3}$?

68. James Wildes bought of John Good,

$45\frac{1}{2}$ bushels Salt, at $39\frac{1}{2}$ cents per bushel;
$143\frac{3}{4}$ lbs. Rice, at $3\frac{3}{4}$ cts. per lb.;
$43\frac{1}{4}$ lbs. Tea, at 39 cts. per lb.;
$94\frac{3}{4}$ lbs. Coffee, at $10\frac{1}{4}$ cts. per lb.;
12 bbls. Flour, at \$5.87 per bbl.

What is the amount of the bill?

69. If he pays the above bill by a bank note, discounted for 60 days, what must be the sum named in the note?

70. Add $23\frac{1}{7}+18\frac{3}{4}+\frac{21\frac{1}{2}}{5}+\frac{6}{13\frac{1}{4}}+\frac{11}{1\frac{1}{2}}$.

71. Reduce to a common denominator, $2\frac{1}{2}$ and $\frac{3}{4}$ of $\frac{7}{11}$ of $12\frac{1}{5}$ and $\frac{8}{9}$ of $\frac{4}{15}$.

72. Bought 3 boxes of sugar, containing 3 cwt. 2

qrs. 17 lbs.; 3 cwt. 3 qrs. $11\frac{1}{2}$ lbs.; 3 cwt. 2 qrs. $22\frac{3}{4}$ lbs.; at $6\frac{7}{8}$ cts. per lb. Paid in corn at $57\frac{1}{4}$ cts. per bushel. How many bushels did it take?

73. What is the solid contents of a wall 5 ft. high, 2 ft. 3 in. wide at the bottom, and 1 foot 10 in. wide at the top, and 34 ft. in length?

74. Three men agreed to build a wall 6 ft. high, 3 ft. wide at the bottom, and 2 ft. wide at the top. The first man built the wall to the height of 2 feet from the ground, the second raised it 2 feet more, and the third finished it. What proportional share of the pay ought each to receive?

75. There is a lever, 13 ft. in length, which is supported by a fulcrum 14 in. from the end. How many pounds applied to the longer end will balance a weight of 17 cwt. 2 qrs. at the shorter end?

76. The axle of a wheel is 13 in. in diameter, the wheel is $11\frac{1}{2}$ ft. in diameter. What power, applied to the circumference, will balance $3\frac{1}{2}$ tons suspended at the axle?

77. If the threads of a screw are $1\frac{1}{4}$ in. apart, what power, applied at the end of a lever $9\frac{1}{2}$ ft. in length, will support 7 tons, allowing nothing for friction?

78. With the same screw, what power would support 7 tons, making an allowance of one fourth for friction? Reflect whether the friction in this case is in favor of the weight or in favor of the power.

79. With the same screw, what power would raise 7 tons, allowing one fourth for friction? Notice, in this case, in which way the friction will operate.

80. There are two right-angled triangles upon a base of 21 ft. in length; the perpendicular of the larger is 9 ft. in length, that of the smaller $8\frac{1}{4}$ ft. What is the difference in the length of the hypotenuse of the two triangles?

81. Divide the number 78 in two such numbers that the first shall be 6 more than one fifth part of the second.

82. Divide the number 82 into two such parts that the first diminished by 5 shall equal one sixth part of the second.

83. What is the value, in Federal money, of £194 16s., at $8\frac{1}{4}$ per cent. advance?

84. How many Winchester gallons, wine measure, would be contained in a cubical vat measuring 4 ft. each way?

85. Divide the number 1520 into three such parts that twice the first shall be 40 less than the second, and the second shall be half as great as the third.

86. How many yards of lining, $\frac{5}{8}$ of a yard wide, will line $13\frac{1}{4}$ yds. of cloth $1\frac{7}{16}$ yds. wide?

87. There is a rectangular field containing $7\frac{1}{2}$ acres, the width of which is one half as great as its length. What is the length of a diagonal line connecting its opposite corners?

88. Around a rectangular common, containing 18 acres, the length of which is to its breadth as 6 to 5, a road runs 40 ft. in breadth. How many rods would be saved in travel by crossing the common diagonally, rather than going round on two sides of it, supposing the traveller to begin and end, in both cases, in the middle of the road in range with the diagonal line?

89. What is the difference between the square root of half of 4, and half the square root of 4, carried to three decimal places?

90. What is the difference between the square root of one third of 12, and one third the square root of 12, carried to three places of decimals?

91. How many times $\frac{3}{4}$ of $17\frac{1}{5}$ are equal to $13\frac{1}{2}$ times $15\frac{2}{7}$?

92. When it is noon in Boston, Lon. 71° 4′ W., what time is it at Liverpool, Lon. 2° 59′ W.; at Greenwich, Lon. 0; at Havre, Lon. 0 16′ E.; and at Paris, Lon. 2° 20′ E.?

93. When it is noon at London, Lon. 0 5′ W., what time is it at New York, Lon. 74° 1′ W.; at Washington, Lon. 77° 2′ W.; and at Cincinnati, Lon. 84° 27′ W.?

94. A field, in the form of an equilateral triangle, contains $8\frac{1}{2}$ acres. What is the length of one of its sides?

95. Reduce $\frac{3}{7}$ of $\frac{8\frac{1}{2}}{11}$ to fifths. Reduce $\frac{4}{5}$ of $\frac{7}{18\frac{1}{4}}$ to fifteenths.

96. What is the amount, in Federal money, of 6s. 7d., 5s. 3d., 14s. 9d., and 11s. $6\frac{1}{2}$d.?

97. What is the cube root of 144, to three places of decimals?

98. What is the weight of a cylinder of lead 3 ft. long and 4 inches in diameter?

99. Multiply $7\frac{1}{2}$ times $\frac{13\frac{1}{4}}{15}$ by $\frac{1}{8}$ of $\frac{34\frac{1}{2}}{39}$.

100. How many square feet in a triangle whose base measures 45 feet, and whose height is 17 feet?

101. What is the solid contents of a triangular pyramid whose base measures 18 feet on each side, and whose height is 23 feet?

102. What is the interest of $546.25 for 23 mo. 13 days, at $7\frac{1}{2}$ per cent.?

103. What number is that of which $\frac{1}{4}$ and $\frac{1}{3}$ of it added together exceed $\frac{1}{2}$ of it by 2?

104. What is the cube root of 197 to two decimal places?

105. What is the cube root of 501 to four decimal places?

106. What is the 43d term of an arithmetical series whose first term is $7\frac{1}{2}$ and common difference $3\frac{2}{3}$?

107. What is the sum of an arithmetical series of 57 terms whose 4th term is 15 and common difference $2\frac{1}{2}$?

108. How many pounds of coffee at 11 cents per

pound can be mixed with 56 lbs. at 8 cents, and 96 lbs. at $9\frac{1}{2}$ cents, so as to make the mixture worth $10\frac{1}{4}$ cents?

109. If a sphere of gold weigh 36 oz., how many ounces will a sphere of silver weigh of equal size, the specific gravity being as given on page 258?

110. What will be the weight of a ball of iron 6 inches in diameter?

111. What will be the weight of the largest cube that can be cut from a ball of iron 6 inches in diameter?

112. What is the value, in Federal money, of £13 6s., 2 guineas, 3d., $7\frac{1}{4}$d., added together?

113. How many times will a wheel 4 ft. 3 in. in diameter go round in traversing the circumference of a circle containing 5 acres?

114. If a lever is 16 ft. in length, the weight 13 cwt. 3 qrs. 11 lbs., and the power 94 lbs., what must be the distance of the fulcrum from the weight in order that the weight and power may balance?

115. If the longer arm of a lever be 10 feet and the shorter arm 2 feet in length, how must 480 pounds be divided so that one part shall be the weight and the other the power that will balance it on the lever?

116. Divide $\frac{3}{4}$ of $\frac{7}{8}$ of $16\frac{1}{5}$ by $18\frac{1}{3}$ times $\frac{2}{7}$ of $7\frac{1}{4}$.

117. Reduce 1 pk. 3 qts. 1 pt. 1 gill, to the decimal of a bushel.

118. How many shillings and pence in .4562 of a £?

119. A general drew up his army in a square, with the number in rank and file equal, and had 576 men left. He then increased the square by placing two lines of men in front, and two files on one side from front to rear, when he found he wanted 12 men to complete the square. How many men had he?

120. What number is that one third of which exceeds two sevenths of it by 19?

121. What number is that $\frac{5\frac{1}{3}}{13}$ of which exceeds $\frac{9}{28}$ of it by 31?

122. How many tiles, each 8 inches square, will it require to cover one acre?

123. If the hypotenuse of a right-angled triangle measure 34 feet, and the base $19\frac{1}{2}$, what is the measure of the perpendicular?

124. If the sum of the base and hypotenuse is 63 feet, and the perpendicular 14 feet, how long is the base?

125. A man travels south 20 miles, then east 15 miles, then south $2\frac{1}{2}$ miles, and east 7 miles. How far is he then from where he set out?

126. What is the difference between the cube root of one third of 12, and one third of the cube root of 12?

127. What is the value, in dollars and cents, of £94 16s. 3d. + £43 19s. 7d. + £14 13s. 9d.?

128. If a note of $1000, promising annual interest, is renewed at the end of each year for five years, without the payment of any interest, what is the amount, principal, and interest, at the end of the fifth year?

129. What is the difference, in avoirdupois weight, between a ball of silver and a ball of gold, each 3 inches in diameter?

130. There are three numbers; the first plus 4 is equal to one sixth of the second, the second is one half as great as the third, and the sum of the three is 186. What are the numbers?

131. There are two numbers; the first increased by 4 equals one sixth of the second, and the second diminished by 6 is eight times the first. What are the numbers?

132. What is the value, in Federal money, of 13 shares of bank stock, par value $125 per share, and sold at 11¼ per cent. advance?

133. What is the square root of 14734?

134. What is the interest of $1974.36, for 3 yrs. 2 mo. 17 days, at 5¾ per cent.?

135. What is the 14th term of a geometrical series, the first term of which is 4 and the ratio ¾?

136. What is the 16th term of a geometrical series, the first term of which is 7 and the ratio 1½?

137. A sells to B 5 loads of wood, measuring, first, 8 ft. 6 in., 4 ft., 3 ft. 9 in.; second, 9 ft., 4 ft. 2 in., 3 ft. 11 in.; third, 9 ft. 2 in., 4 ft. 4 in., 4 ft. 1 in.; fourth, 8 ft. 2 in., 3 ft. 7 in., 4 ft. 2 in.; fifth, 7 ft. 11 in., 4 ft., 3 ft. 6 in.; at $4.75 per cord. He receives in payment 47 bushels of oats, at 38 cents per bushel; 56 lbs. of cheese, at 8½ cents per pound; and the balance in butter, at 15½ cents per pound. How much butter does he receive?

138. What is the weight of one rod of lead pipe one fourth of an inch in thickness, if the inner diameter measures 1¼ inches?

139. What is the weight of a plate of iron half an inch in thickness, 4 feet long, and 2 feet 3 inches in breadth?

140. How many feet of silver wire, one tenth of an inch in diameter, can be made from one pound avoirdupois of silver?

141. How many gallons, imperial measure, will a cylindrical cistern hold, 3 feet in diameter and 4½ feet deep?

142. What is the cost of transporting 64 barrels of flour, each containing 7 qrs. gross, 100 miles, at $3.12½ per ton, allowing for the weight of each cask 16 lbs.?

143. If freight by railroad is $3.12½ per ton, for 100 miles, and freight by wagon road is $20 per ton for 80 miles, how much is saved in the freight of a

barrel of flour 100 miles by railroad, allowing its weight to be as in the preceding example?

144. At the rate named above, how far could a barrel of flour be carried by wagon road, before the freight should amount to as much as the flour was worth, when the price is $5.62 per barrel?

145. If the distance from New York to Liverpool is 3000 miles, what would be the cost of transporting a barrel of flour that distance, at the rate of $20 per ton for every 80 miles?

146. If a barrel of flour can be transported from New York to Liverpool for 65 cents, what would that give for the transport of one ton 80 miles?

147. The summit of the Rocky Mountains, visited by Freemont, is in longitude 110° 8′. What time is it at Greenwich when it is noon there?

148. There is a field 20 rods long and 8 rods broad, with a path 3½ feet wide running round it. How many square feet are there in the path?

149. What is the cost of excavating a cubical pit, measuring 7¼ feet in each direction, at 31¼ cents per cubic yard?

150. What is the weight of an iron cannon 9½ feet in length, 26 inches in diameter at the larger end, and 18 inches at the smaller, with a bore 9 feet long and 10 inches in diameter, allowing for no inequalities in the surface?

151. What will be the duties on an invoice of goods amounting to $1156.80, at 30 per cent.?

152. Bought an invoice of imported goods, amounting to £564 15s. I agree to give 18 per cent. in advance of the invoiced price. What is the amount paid, in Federal money, reckoning $4.444 to the pound?

153. I buy for cash an invoice of imported goods, amounting to £1146 16s. What is the invoiced price,

in Federal money, allowing Sterling money to be 9 per cent. in advance of the nominal par value?

154. A merchant owes $16472.50. He fails, his means of payment amounting to only $4345.62. How much is a creditor entitled to, who holds a note against him of $100, dated $7\frac{1}{2}$ months previous to the final settlement, and promising interest 60 days after date?

155. A road, $3\frac{1}{2}$ rods wide, is laid out 1 mile and $14\frac{1}{2}$ rods in length, for which damages are awarded to the land-owners, as follows:—for 80 rods of the road, at the rate of 37 dollars per acre; for 110 rods, at the rate of 22 dollars per acre; and for the remainder, at the rate of $30\frac{1}{2}$ dollars per acre. What is the whole amount of the damages?

156. What is due for the freight of 12 barrels of flour 65 miles, at $\frac{5}{8}$ of a cent per pound, allowing each barrel to contain 7 qrs. gross, of flour, and the cask to weigh 18 pounds?

157. If 5 barrels of flour suffice for a family of 11 persons 7 months, how many barrels will suffice for a family of 15 persons $4\frac{1}{2}$ months?

158. $13\frac{1}{2}$ times $14\frac{4}{5}$ is $7\frac{1}{4}$ times what number?

159. 384 is $\frac{7\frac{1}{4}}{9}$ of how many times $15\frac{1}{7}$?

160. Reduce $62\frac{1}{2}$ cents to the decimal of a £, at nominal par value.

161. How many seconds were there in the year 1844?

162. The report of a signal gun, fired on the equator, at 12 o'clock, is heard at a place due west distant 16 miles. At what time is the report heard at the latter place, allowing $69\frac{1}{2}$ miles to a degree of longitude, and sound to move 1 mile and 10 rods in 5 seconds?

163. A communication is made by the magnetic telegraph from Boston to Washington, at 1 o'clock,

P. M. At what time will it be received at Washington, allowing no time for the transmission of the fluid; longitude of Boston being 71° 4′ 9″; that of Washington, 77° 1′ 24″?

164. A, engaging in partnership with B and C, puts in 1600 dollars for $9\frac{1}{2}$ months; B puts in 3100 dollars for 14 months; C puts in 2200 dollars for 12 months. They gain 1046 dollars. What is each one's share of the gain?

165. If 100 dollars in one year gain $6\frac{1}{2}$ dollars interest, what will 467 dollars gain in $9\frac{1}{2}$ months, at the same rate?

166. What is the value of 17 shares of bank stock, par value 60 dollars per share, and sold at $6\frac{1}{4}$ per cent. advance?

167. A man buys 640 barrels of flour at $\$5\frac{3}{8}$ per barrel; pays for freight, $42.75; for storage, $11.17; and sells it for $6\frac{5}{16}$ per barrel, allowing a commission of $2\frac{1}{4}$ per cent. What was his loss or gain per cent.?

168. The pipe of an aqueduct, 12 inches in diameter, is divided into two branches, such that their united capacity is equal to that of the main pipe, and the diameter of one of the branches is 9 inches. What is the diameter of the other?

169. The pipe of an aqueduct is 3 feet in diameter. What number of pipes, each $2\frac{1}{2}$ inches in diameter, would have a capacity equal to that of the main pipe?

170. If a tree measures, at the distance of 2 feet from the ground, 12 feet in circumference; and, at 14 feet from the ground, divides into two branches, measuring respectively 9 feet and 7 feet in circumference; how many square inches more of surface would the lower horizontal section of the tree contain than the upper one?

171. A certain tree measures, at the distance of 3 feet from the ground, $22\frac{1}{2}$ feet in circumference. At some distance above, its four branches measure, re-

spectively, 8 ft. 4 in., 9 ft., 7 ft. 6 in., and 6 ft. 5 in. in circumference. What is the ratio of the magnitude of the tree at the lower, compared with its magnitude at the higher place of measurement?

172. The shadow of a certain tree, as cast upon the ground, measures 102 feet in diameter. Allowing the shadow to be circular, how many rods of ground does it cover?

173. How many cubic inches does a wine-glass contain, measuring $3\frac{1}{2}$ inches in depth, and 2 inches in diameter at the top, the form being that of an inverted cone?

174. How many yards of lining, $\frac{3}{8}$ of a yard wide, will line $37\frac{1}{4}$ yards of cloth $1\frac{7}{16}$ yards wide?

175. What is the 3d term of the square $900+420+\square$?

176. What is the 3d term of the square $1600+480+\square$?

177. Complete the square $4900+\square+64$.

178. What is the 4th term of the cube $27000+5400+360+\square$?

179. What is the 4th term of the cube $64000+4800+120+\square$?

180. What is the 4th term of the cube $125000+22500+1350+\square$?

181. What is the cube root of 68437?

182. What is the cube root of 954326?

183. What is the solid contents of the largest sphere that can be cut from a cubic block $13\frac{1}{2}$ inches on a side?

184. From a sphere measuring 10 inches in diameter, the largest possible cubic block has been cut; and from this block again the largest possible sphere has been cut. What is the diameter of the last-named sphere?

185. From a sphere 20 inches in diameter what are the dimensions of the largest parallel prism that can

be cut, making the length to the breadth as 4 to 3, and the breadth to the thickness as 3 to 2?

186. How many cubic feet are there in the walls of a brick house, the length and breadth of which are each 44 feet outside, and the height 24 feet, supposing the walls to be perpendicular outside, and the thickness to be 2 feet at the bottom and 1 foot at the top, making no deduction for doors or windows?

187. What are the prime factors of 3746? Of 9862?

188. What is the least common multiple of 684, 963, and 8416?

189. What is the greatest common divisor of 94620, 3642, and 1646?

190. How many times will the wheel of a railroad car, if it be $2\frac{1}{2}$ feet in diameter, revolve in going 40 miles?

191. How many times will such a wheel revolve in a minute, if the speed of the car be 20 miles an hour?

192. What is the cost, in Federal money, of the freight of 940 bales of cotton, averaging 340 lbs. per bale, at $\frac{3}{8}$ of a penny per pound?

193. Sold 34 pieces cotton goods, averaging $31\frac{1}{2}$ yards each in length, at $9\frac{3}{8}$ cents per yard, $1\frac{1}{2}$ per cent. off. What is the amount of the bill?

194. Bought 840 barrels of flour, on six months' credit, at $4\frac{7}{8}$ dollars per barrel. Sold the flour the same day on three months' credit, at $4\frac{15}{16}$ dollars per barrel. Did I gain or lose, and how much, estimating the present worth of the debts at the date of the transaction?

195. A merchant buys for me, on commission, 400 barrels of flour, for $4.34 per barrel, cash. He sells the flour on the same day for cash, at $4.46 per barrel. How much do I gain by the operation, allowing $1\frac{1}{2}$ per cent. commission on the purchases, and 2 per cent. on the sales?

196. A sets out on a journey, travelling 24 miles a day. B sets out 2 days after, travelling $31\frac{2}{3}$ miles a day. A, after travelling 3 days, goes 25 miles a day; and B, after travelling 4 days, goes $33\frac{1}{4}$ miles a day. In how many days after A sets out will B overtake him?

197. If a parallel prism is $4\frac{1}{2}$ inches thick, 5 inches wide, and 6 inches long, what must be the diameter of the hollow sphere that would enclose it?

198. If 18 horses, in 16 weeks, consume 204 bushels of oats, how many horses will it require to consume 413 bushels in 18 weeks?

199. If 100 dollars gain 6 dollars interest in 12 months, what will be the interest of 840 dollars for $5\frac{1}{2}$ months?

200. If a field containing 17 acres measures 81 rods on one side, what must be the length of the corresponding side of a similar field containing $26\frac{1}{2}$ acres?

201. The contents of two similar fields are as $4\frac{1}{2}$ to 7, and the smaller measures on one side 63 rods. What must be the corresponding dimension of the larger field?

202. There are two circles; their areas are as 14 to 19, and the diameter of the smaller is 16 rods. What is the diameter of the larger?

203. What is the area of a circle whose diameter is 44 feet?

204. What is the circumference of a circle whose diameter is 67 inches?

205. Given the circumference of a circle 60 rods, to find its area.

206. There are three equilateral triangles, whose areas are to each other as the numbers 3, 4, and 7. A side of the smallest measures 40 rods. What is the sum of their areas?

207. A grindstone is 3 feet in diameter. Allowing the hole in the middle to be 2 inches in diameter, how

many inches must be ground off to grind away half of the stone?

208. The fore wheels of a wagon are 3 feet 10 inches in diameter, and the hind wheels 4 feet 2 inches in diameter. How many times more does one of the fore wheels turn round than one of the hind wheels in going one mile?

209. What is the weight of a cast-iron cylinder 6 feet long and $4\frac{1}{2}$ inches in diameter, the specific gravity being as already given?

210. The frustum of a cone 7 feet long is 14 inches in diameter at the larger end, and 10 inches in diameter at the smaller. How far from the base must it be cut in two to divide it into equal parts?

211. What is the first prime number above 901?

212. What is the first prime number below 10000?

213. What is the greatest common divisor of 1846, 3105, 684, and 1006?

214. What are all the prime factors of 801, of 3042, of 586, of 908?

215. In what proportion may corn at 80 cents be mixed with rye at 86 cents, and with oats at 43 cents, per bushel, to make the mixture worth 50 cents per bushel?

216. Add the fractions $\frac{3\frac{1}{4}}{7}+\frac{42}{61\frac{1}{2}}+\frac{19}{37\frac{1}{3}}$.

217. What is the value of $\frac{3}{7}$ of $\frac{1}{9}$£, $\frac{4}{6}$ of $\frac{3}{10}$ s., expressed in the decimal of a £?

218. What is the present value of a note of 584 dollars, payable in three months?

219. What is the bank discount on a note of 150 dollars, payable in three months?

220. What sum will be paid on a note of 240 dollars, discounted at a bank, for 90 days?

221. What is the interest of 1200 dollars for 10 days, at 7 per cent.?

222. Divide the sum of the decimals 2016 + 9172 + 0064, by $\frac{3}{4}$ of $\frac{5}{8}$ reduced to a decimal.

223. Divide 3 T. 17 cwt. 3 qrs. 19 lbs. by 6.

224. Divide 9 m. 3 fur. 21 r. 14 ft. by 8.

225. Multiply 31 d. 14 h. 37 m. 15 sec. by 19.

226. Multiply 83 A. 3 R. 22 r. by 12.

227. What is one fifth of 16 T. 11 cwt. 2 qrs. 20 lbs.?

228. What is the value, at 4 dollars a cord, of three loads of wood, measuring as follows: first, 8 feet 6 in., 4 ft. 2 in., 3 ft. 9 in.; second, 9 ft., 4 ft. 1 in., 3 ft. 10 in.; third, 8 ft. 1 in., 4 ft., 4 ft. 2 in.?

229. What is the 15th term of an arithmetical series, the 1st term of which is 3 and the common difference $\frac{1}{6}$?

230. If the 9th term of an arithmetical series is 23, and the common difference $\frac{2}{3}$, what is the 3d term?

231. What is the sum of an arithmetical series of 40 terms, if the 1st term is 2 and the common difference $3\frac{1}{5}$?

232. What is the sum of an arithmetical series of 72 terms, if the 1st term is 1 and the common difference $1\frac{3}{7}$?

233. A man engages to walk 1000 miles in 1000 hours, on condition of receiving 1 cent for the first mile, and for each mile after $\frac{1}{4}$ of a cent more than he had for the mile preceding it. What will he be entitled to on the fulfilment of his contract?

234. There are two grindstones, the thickness of which is to the diameter as 2 to 11. The smaller one is 2 feet in diameter; the other is three times as heavy. What is its diameter?

235. A man has a triangular field containing 7 acres. The vertex or point of the triangle is 54 rods from the base. At what distance from the base must a line be drawn parallel to it so as to cut off one half the field?

236. The hypotenuse and perpendicular of a tri-

angle measure together 816 rods; the base measures 61 rods. What is the length of the perpendicular?

237. A rope 100 feet long passes straight from the ground, at the distance of 10 feet from a perpendicular pole, over the top of the pole, which is 25 feet in height, and thence is drawn so as to reach the ground at the farthest possible point. Allowing the ground to be level, how far is the last-named point from the foot of the pole?

238. A stick of timber, in the form of a truncated wedge, is 10 feet long, 2 feet wide through its whole extent, 20 inches in thickness at one end, and 14 inches thick at the other. How far from the thicker end must it be cut in two so as to divide it into two equal parts?

RECOMMENDATIONS.

From Mr. George B. Emerson, Boston.

I have carefully examined the plan of Mr. Adams's work on Mental Arithmetic, and have given some attention to its execution; and I am confident that it will prove a very valuable addition to the means of instruction in Arithmetic. It is a successful extension of the admirable method of Colburn's First Lessons, with such modifications as seemed to be required in a higher work on the same general model. It occupies unappropriated ground; and it deserves, and I think it will take, a high place amongst the text-books.

GEO. B. EMERSON.

From Mr. Thomas Sherwin, Boston.

I have carefully examined, in manuscript, the work of Mr. Adams on Mental Arithmetic, and am much pleased with it. His plan is good, and well executed. I would, therefore, heartily recommend his book to Teachers and School Committees, as one which will contribute very materially to the attainment of that very important, but much-neglected, branch of study, — Intellectual Arithmetic.

THOMAS SHERWIN,
Principal of the Boston English High School.

From Mr. Solomon Adams, Boston.

To the Publisher.

Dear Sir: — Having been favored with an opportunity of examining, in manuscript, a Treatise on Mental Arithmetic, by F. A. Adams, A. M., I am most happy to find that our schools are about to have a work of the kind, carried with much skill and judgment into the higher departments of Arithmetic.

Very respectfully yours,
SOLOMON ADAMS,
Principal of the Young Ladies' School, Central Place.

From Roger S. Howard, Esq., Newburyport.

MR. F. A. ADAMS.

DEAR SIR: — I have looked over, with much care and pleasure, the manuscript Arithmetic, which you put into my hands a few days since. The plan of the work appears to me quite original, and many of the methods you have adopted exceedingly ingenious, and, at the same time, beautifully simple. Your rules and explanations are clear and concise; and the numerous examples for practice which you have inserted, are judiciously selected and well arranged. The book, I think, is one which will greatly facilitate the teaching of this important branch of education.

I am, sir, very respectfully yours,

ROGER S. HOWARD,

Principal of the Putnam High School.

From Mr. Rufus Putnam, Salem.

MR. F. A. ADAMS.

DEAR SIR: — I have read with much satisfaction the manuscript copy of the Mental Arithmetic you are intending to publish. The plan of the work is, in many respects, different from its predecessors; and, strange as it may seem, to those who examine many of the new books in the various departments of education, and who have not read yours, it occupies much ground which has not been occupied by others. I think that, in its arrangement, its definitions, its explanations, the examples for practice, — indeed, in its whole matter, — it is happily adapted to its object; to release our youth from a part of their present bondage to slate and pencil, and artificial rules, by qualifying them to perform correctly and easily, *in the mind*, many of the operations which are almost universally performed on the slate. I commend it, with much confidence, to the notice of all who are intrusted with the education of youth.

Yours, very truly and respectfully,

R. PUTNAM,

Principal of the Bowditch (English High) School.

From Mr. Edwin Jocelyn, Salem.

MR. F. A. ADAMS.

DEAR SIR: — No one can hold "Colburn's First Lessons in Arithmetic" in higher estimation than I do; and I think, whoever undertakes to furnish a substitute for that little book, which shall better answer the purpose, will fail in *his* purpose. I am glad to see from your hand an *extension* of Mental Arithmetic on the plan of that inestimable school-book. I have often felt the want of such a work, and have in practice extended this course of teaching, somewhat: — and should have done it oftener and farther, if I had had such a book at hand as you now propose to publish. The plan appears to me to be very happily carried out, and I feel confident that it will meet with a wide appreciation and use.

Yours, with much esteem,

EDWIN JOCELYN,

Principal of the Female High School.

From Mr. Charles Northend, Salem.

Mr. F. A. Adams.

Dear Sir: — Having examined, with some care, your manuscript entitled "Advanced Lessons in Mental Arithmetic," I feel no hesitation in saying, that I consider the work very happy in design, and admirable in execution. Wishing you much success,

I remain very truly yours, &c.,

CHARLES NORTHEND,

Principal of Epes Grammar School.

From the North American Review.

To the late Warren Colburn belongs the high credit of first introducing into our Schools, through his admirable First Lessons, the regular study of Mental Arithmetic. Of this excellent little manual, the author of the book before us justly observes, that, so completely has it performed the work within its prescribed sphere, that there is little reason to desire or to expect that it will ever be superseded. Mr. Colburn published also a Sequel to Mental Arithmetic, in which the principles and rules of Written Arithmetic were deduced from the solution and analysis of questions according to the method adopted in the former treatise. This Sequel was very well executed as far as it went; but it was not full enough for all the wants of the higher classes in our schools. It omitted Proportion and Progression, the "Rule of Three," and the doctrine of Powers and Roots. Mr. Adams has undertaken to supply these deficiencies, following mainly in the track of Mr. Colburn, but appearing fully competent also to mark out a path for himself. By this enlargement of plan, he has brought many useful problems in Mensuration and Mechanics within the scope of his work, and has extended the analysis and induction over much new ground, though many of the new problems are still left to be performed by arbitrary rules.

The First Part of Mr. Adams's book consists of exercises in Mental Arithmetic, arranged under the different arithmetical rules. Where the principles have not been taught in the First Lessons, they are here carefully deduced from an analysis of a number of simple questions, following which are numerous and well-selected examples. These examples pass gradually from simple to more complicated questions, so as to give the pupil a thorough training. In the Second Part, the different processes are arranged in the same order as before; and when the operations are complicated, distinct rules are given, illustrated by examples for practice containing larger numbers than were suitable for the exclusively mental operation. When the operations are simple, and sufficiently explained in the analysis and induction contained in the First Part, a reference is merely made to that Part, and the examples for practice follow, without any enunciation of a rule.

The author's reasoning and explanations are very clear, simple, and concise; his disposition of the different parts judicious, and his selection of examples well suited to exercise the mind of the pupil. As a whole, we prefer this work to any Arithmetic we have seen in use.

From Professor Chase, of Dartmouth College.

MR. F. A. ADAMS.

MY DEAR SIR: — I have examined, with some care, your Treatise on Arithmetic, and am much pleased with it. The practice and habit of extending mental operations to large numbers, is of great utility. I have occasion, very frequently, to see the inconvenience that young men suffer, from the want of such a habit. Not less valuable than the habit of operating mentally upon large numbers, is the habit of performing the more advanced operations of arithmetic without the aid of the pencil.

I like very much, also, the manner in which you have treated several of the principles which you have developed; as, for example, the subject of the Common Divisor, the Least Common Multiple, the Roots, Ratio, and Proportion. These are but few of the subjects, but I mention them as examples.

I think the book will do much to promote the proper method of teaching arithmetic, — by *demonstration* and explanation.

I am, dear sir, very truly yours, &c.,

S. CHASE.

From Mr. Addison Brown, of Brattleboro, Vt.

DANIEL BIXBY, ESQ.

DEAR SIR: — I give you many thanks for the "First Book in Arithmetic," by F. A. Adams, which you had the kindness to send me, some time since. To test its merits, I set my youngest child, a daughter, seven years of age, to studying it. She has been about half through it; and having heard her lessons myself, and watched her progress, and the effect the book has had in developing her powers of calculation, I am satisfied that it is a very excellent work. I think, as a First Book in Arithmetic, it is decidedly the best I have either used or examined, and I should be glad to see it extensively introduced into our Schools.

Yours, with respect,

ADDISON BROWN.

From Mr. John Tatlock, Professor of Mathematics, and Mr. A. Hopkins, Professor of Natural Philosophy.

I have examined a treatise on Arithmetic, by F. A. Adams, and am much pleased with it. I think it well adapted to teach the science and art of numbers, and at the same time to teach the art of thinking. I am persuaded that a thorough training in this Arithmetic would prepare students for the further study of mathematics better than nine-tenths are now prepared.

I should be glad if every student who enters college was master of this Arithmetic.

JOHN TATLOCK.
A. HOPKINS.

From Mr. Roger B. Hildreth, Tyngsborough.

To the Publisher.

Dear Sir: — The "Advanced Lessons in Mental Arithmetic," upon the plan adopted by Mr. F. A. Adams, will, I am persuaded, from a careful examination of the work, be found very useful in instruction. It peculiarly meets the wants both of the teacher and the pupil; and as its real merits become more generally known, I am confident it will be extensively used as a text-book. The work is arranged with admirable clearness, — the rules and explanations are concise, and yet very simple. I cheerfully commend it to the notice of all who wish for an excellent treatise on Mental Arithmetic.

Very respectfully yours,

ROGER B. HILDRETH,

Principal of Tyngsborough High School.

From J. H. Purkitt, Esq., St. Louis.

Mr. Bixby.

Dear Sir: — I have used Frederic A. Adams's Arithmetic in my School for many months; and it gives me pleasure to say that it has met my highest expectations. I regard it as by far the best Arithmetic in the English language. Recommendations are extremely cheap, nor would I trouble you with an expression of my own opinion of its merits, did I not believe it to be an act of common justice, when an author has produced a really valuable book, that those who have *thoroughly* tested it, and *know* and *feel* its value, should freely and cordially acknowledge it. If it has a decided superiority over all other Arithmetics, in developing power of thought, and training the mind to a stern and rigid analysis, then the public have a right to know it. That this book, faithfully used, will do this, I sincerely believe. No consideration whatever could induce me to throw it out of my school, for the sake of introducing any other Arithmetic yet published. I feel, therefore, that I am doing not only what is just, but what, I hope, is a real good, by this expression of opinion. Teachers may be slow in using it as a text-book in their schools; but that it will be ultimately adopted, and, when adopted, permanently retained, I cannot, for a moment, doubt. To the lazy and unthinking, and those who are bigotedly attached to the common mode of teaching Arithmetic, this book will not only not be acceptable, but, perchance, be dogmatically condemned, and indignantly rejected. But those who love to think themselves, and desire to develope the power of correct thinking in others, will find in this book an exhibition of principles, beautiful as they are true, and ingenious as they are rational. Wishing you great success,

I am, dear sir, yours, with respect,

J. H. PURKITT,

Principal of the Young Ladies' High School.

From Mr. Charles C. Dame, Principal of the English High School, Newburyport.

MR. F. A. ADAMS.

DEAR SIR: — I have examined somewhat carefully your "Mental and Written Arithmetic," and find that it possesses *peculiar merit.* The arrangement and exercises are such, I think, as will train the student to a habit of thinking and reasoning for himself, and lead him readily to apprehend the relations of numbers in their various combinations. The book is not encumbered with "many rules," but the subjects of which it treats appear to be taken up in a systematic manner; and the explanations are clear and analytic.

The work throughout seems to have been planned and executed in a way well calculated to fit those who may use it for the "active pursuits of life,"— to enable them to solve, in the most ready and natural way, those arithmetical questions which may occur in business or otherwise. The questions for exercise which it contains are, for the most part, practical, and so constructed as to afford encouragement in their solution.

I have formed such an opinion of the work, as to believe that no scholar will hereafter leave school where it is known, and consider himself well educated, unless he is familiar with its pages, or the principles which they contain.

I am very truly and respectfully yours,

CHARLES C. DAME.

From E. Wyman, Esq., St. Louis.

MR. F. A. ADAMS.

DEAR SIR: — It is now nearly two years since I introduced your Arithmetic into my School, and I have refrained from expressing any opinion of its merits, until it should have been fairly and thoroughly tested. I did not introduce it — nor do I any book — simply as a matter of *experiment.* I *believed* it to be a good work, from *examination* of it; and am now prepared to add, that I *know* it to be a good one, from *trial.*

I cheerfully assent to the opinions I see expressed by others, on all the valuable characteristics of the book. Beside these, I must mention the fact, that upon examination of those students who have been carried through the system, and made to comprehend it, I find them, in their arithmetical processes, independent of arbitrary diction of rule, possessing a strong analytical power, and arriving at results with great rapidity and accuracy. The head-work takes precedence of the hand-work; and they have a *why* and *wherefore* for what they do. My commendation of the book is full and unequivocal.

Yours truly,

E. WYMAN, A. M.,

Principal of the St. Louis English and Classical High School.

From Elias Nason, A. M., Principal of the Classical High School, Newburyport.

MR. ADAMS.

DEAR SIR:—Having carefully examined your treatise on Arithmetic, I would say that I have formed a very favorable opinion of it, and believe it to be a valuable contribution to the department of instruction to which it relates. Your method of performing mental operations on large numbers is philosophical, and, for the most part, original; your rules and illustrations are written with clearness and precision, and your examples for practice have been chosen with reference to the actual concerns of life. You have not encumbered the book with arithmetical puzzles, which tend to perplex the pupil, and occupy time which should be devoted to useful problems. The practical and business-like character of the work is, in my opinion, a great excellence. The examples are such as are of daily occurrence in the family, the work-shop, and the counting-room; and the scholar who has become master of them will not need another course of training, and a new set of rules, to meet the actual wants of trade and business in the world.

Your method of working Fractions, both Vulgar and Decimal, is rational and easy; and the doctrines of Proportion and of the Roots are plainly and distinctly unfolded.

The exposition of the laws of Mechanics, and the sections on the Comparison of Similar Surfaces and Solids, on Per Centage, Mensuration, and on Weights and Measures, are exceedingly valuable.

In short, I cannot but think that you have presented the whole science of arithmetic in a very interesting and philosophical point of view; and I believe that those teachers who use your book will find it admirably fitted both to develop the intellectual powers of their pupils, and to make them quick and accomplished arithmeticians.

With respect, I remain yours,

ELIAS NASON.

From Mr. William Smyth, Professor of Mathematics, Bowdoin College.

I have examined the system of Arithmetic by the Rev. F. A. Adams, Principal of Dummer Academy. The plan of the work, and the style of its execution, appear to me well calculated to give to the learner clear views of the general principles and operations of Arithmetic, and to furnish the discipline requisite to a skilful and ready application of them. The work, indeed, as should be the case in all works of the kind, appears to have been composed in the recitation room, by one well conversant with his subject, and possessing, in an eminent degree, the talents requisite to a successful instructor; and is therefore admirably adapted to the wants both of the pupil and teacher. I should regard with much pleasure its extensive introduction into our schools and academies.

WM. SMYTH.

From Mr. John D. Philbrick, Principal of the Mathematical Department of the Mayhew School, Boston.

MR. F. A. ADAMS.

MY DEAR SIR: — I am delighted with your Arithmetic. A careful examination of every page of it has convinced me that it is a work of transcendent excellence. To say that it contains a great amount of matter well arranged; that its rules and explanations are clear and logical, and the examples well adapted to illustrate them, would be to accord to it but a small part of its just meed of praise.

Its peculiar and crowning merit is, that it is calculated to emancipate the learner from the bondage of rules, and even to give him dominion over them, so that they shall be to him as clay in the hands of the potter. I cannot but regard it as a superstructure worthy of its admirable basis, Colburn's First Lessons; and if the one be a "faultless" school-book, the other is not a whit less perfect. I am confident, therefore, that it needs no other recommendations than its own merits, to insure it a hearty welcome every where among intelligent teachers.

Yours truly,
JOHN D. PHILBRICK.

From Mr. Stephen Holman, Principal of Fitchburg Academy.

MR. F. A. ADAMS.

DEAR SIR: — I have examined your work on Arithmetic, and find it admirably adapted to teach the "science of numbers." It will be received as a valuable assistant, by those teachers who aim to give their pupils a thorough knowledge of the properties of numbers, as it enables the learner to derive much from the text-book, which has heretofore been communicated orally, if at all. The scholar who studies this book faithfully, will see the subject divested of its mystery, will learn to worship rules less, and common sense more.

I have seen no Arithmetic so well adapted to the wants of our schools.

I am, sir, very respectfully yours,
STEPHEN HOLMAN.

From Mr. A. K. Hathaway, Principal of the Grammar School, Medford.

MR. ADAMS.

DEAR SIR: — I have very carefully examined the "Advanced Lessons in Mental Arithmetic," and with the highest satisfaction. The plan of the work is admirable, and the execution shows great care and judgment.

This work occupies new and unappropriated ground, and is just the manual most needed in our schools, to give that high degree of mental discipline, so necessary in training the young. It needs only to be known to be appreciated; and, when once known, I am fully confident it will be very extensively used.

Yours very truly,
A. K. HATHAWAY.

From Rev. James Means, Principal of Lawrence Academy, Groton.

Daniel Bixby, Esq.

Dear Sir: — I have carefully examined the "Mental and Written Arithmetic" which you put into my hands, and am prepared to give it my unqualified approval.

It would be inappropriate to detail all the excellences which it possesses. I will just mention, as points which gratify me, the method of multiplying and dividing large numbers, used by all good accountants, and now introduced for the first time into a school-book; the manner of stating and illustrating the rules; the nature of the questions proposed for solution; the introduction of matter quite new upon several points of practical importance. I would refer to the rules for Compound Proportion, Square and Cube Roots, several parts of Section XLIV., and all of Section XLVI., as worthy of special notice.

I hope and believe that this book will be extensively used. I shall commend it to the Committee of the Trustees here, who have such matters in charge, and make no doubt it will be permanently established as a text-book.

Very respectfully yours,

JAMES MEANS.

From Teachers of the Public Schools in Lowell.

Having carefully examined the Mental and Written Arithmetic by F. A. Adams, we do not hesitate to say, that, in its design "to continue and extend the course of discipline in numbers," it is, in our opinion, far superior to any thing that has fallen under our notice.

CHARLES MORILL,	*8th*	*Grammar*	*School.*
NASON H. MORSE,	*4th*	*do.*	*do.*
O. H. MORILL,	*6th*	*do.*	*do.*
JONA. KIMBALL,	*3d*	*do.*	*do.*
PERLEY BALCH,	*1st*	*do.*	*do.*

From the School Committee of Lowell.

At a meeting of the School Committee of Lowell, held March 15th, 1847, it was

Voted, That F. A. Adams's Arithmetic be adopted for the High School.

FREDERICK PARKER, *Secretary.*

At a meeting of the School Committee of Lowell, held May 2d, 1848, the principals of several of the Grammar Schools having expressed a desire to use F. A. Adams's Arithmetic in their schools, on motion, it was

Voted, That F. A. Adams's Arithmetic be adopted, to be used as a text-book in any Grammar School the principal of which may be desirous of using it.

FREDERICK PARKER, *Secretary.*

From Mr. A. Parish, Principal of Springfield High School.

MR. F A. ADAMS.

DEAR SIR :—I have just completed the perusal of your Arithmetic, designed to impart to pupils the power of solving, *mentally*, problems involving large numbers. The misgivings which I felt at the commencement of my investigation vanished as I advanced. While the leading feature of the work resembles, in some respects, Colburn's unrivalled production, in its processes and application of principles, it seems to open to the pupil an entirely new field for mental calculations. The great facility with which large numbers are rapidly disposed of without encumbering the mind, is an element in the work which must contribute greatly to the satisfaction as well as success of the scholar.

The very clear explanation and illustration of principles; the *original* and varied character of the problems, happily graduated from the simple to the more difficult; the adaptation of the whole matter, both to produce mental discipline and due preparation for business, must commend the book forcibly to the attention of teachers who desire to employ the most effectual aids in their profession.

A. PARISH.

THE

HISTORICAL SCHOOL PUBLICATIONS

OF

THOMAS, COWPERTHWAIT & CO.

ARE VERY NUMEROUS, AND COMPRISE MANY WORKS OF RARE MERIT.

LORD'S HISTORY.

A Modern History,

FROM THE TIME OF LUTHER TO THE FALL OF NAPOLEON.

FOR THE USE OF SCHOOLS AND COLLEGES.

BY JOHN LORD, A.M.,

LECTURER ON HISTORY.

Of the fitness of Mr. Lord to prepare such a history, some opinion may be formed from a perusal of the English and American testimonials of his Historical Lectures, a few of which are appended.

ENGLISH TESTIMONIALS.

From the Citizens of Edinburgh.

To the Rev. John Lord, A. M., of the United States.

Rev. Sir,—Having learned that you are at present visiting this city, and having heard of the Lectures which you have recently delivered in London and other places in the South, on "Monastic Institutions," "the Reformation and its consequences," and other kindred topics, and being aware of the very high approval with which these Lectures have been received by the most competent judges, we earnestly request you to afford to the inhabitants of this city the advantage which has been reaped by our friends in the South, by delivering, in some suitable place, these Lectures, or such portion of them as you may judge advisable.

THOMAS CHALMERS, D.D., LL.D.
JOHN BROWN, D.D.
WILLIAM CUNNINGHAM, D.D.
THOMAS J. CRAWFORD, D.D.
W. LINDSAY ALEXANDER, D.D.
JAMES GRANT, D.D.
ROB. S. CANDLISH, D.D.

And twenty-three other Professors and Ministers.

Resolutions offered in Boston, January, 1849.

Rev. S. K. Lothrop, after a few introductory remarks, offered the folowing resolutions:

Resolved 1*st,* That we have listened with great interest and satisfaction to the course of very able and instructive Historical Lectures delivered by Mr. Lord, and completed this evening.

Resolved 2*d,* That the subjects selected for these Lectures were, in our judgment, admirably adapted to illustrate the important principles involved in some of the great social struggles of the Middle Ages—a portion of history too much neglected—and were discussed by the lecturer in a clear, yet condensed and comprehensive style, with great vigor and discrimination of thought, and with broad, impartial, and just reflections.

Resolved 3*d,* That Mr. Lord's enthusiastic devotion to historical studies, and his large acquisitions in this department of learning, eminently qualify him to be an able and instructive Teacher of History, and entitle him to the respect of scholars and the encouragement of the community.

DANIEL SHARP, *Chairman.*

Thos. B. Hall, *Secretary.*

From W. B. Hodgson, LL.D., *Principal of the Mechanics' Institute, Liverpool.*

The Rev. John Lord has just delivered at this institution a course of Lectures on the Middle Ages. These Lectures have throughout attracted large audiences, who have listened with respectful attention and marked delight to Mr. Lord's eloquent and just delineations of the institutions and customs of European society, during a period intensely interesting to the philosophical student of History.

W. B. HODGSON,
Principal of the Mechanics' Institute, Liverpool.

From the Citizens of New Haven.

At the close of the Lectures by the Rev. Mr. Lord, the following resolutions were proposed and unanimously adopted:

Resolved, That we have listened with great interest to the elegant and instructive course of Lectures on the Middle Ages, and that their uncommon value induces us to desire that a greater portion of our community may have an opportunity of hearing them.

Resolved, That Hon. A. N. Skinner, Rev. Dr. Bacon, and Professor D. Olmstead, be a committee to express our thanks to Mr. Lord, for the pleasure and instruction which his Lectures have afforded us, and to ascertain whether he can be induced to repeat the course for the benefit of many persons who are desirous of hearing them.

From the Faculty of Union College.—May 15, 1849.

The Rev. John Lord has, by request, recently delivered six of his Historical Lectures before the senior class in this College. The undersigned have heard most of them, and take pleasure in expressing their approbation of the manner in which Mr. L. has treated some of the most interesting and difficult points and characters of European history. His knowledge is extensive, his views impartial and comprehensive, and his style of lecturing exceedingly well fitted to guide the studies of youth in the important branch of literature to which he is devoted.

ELIPHALET NOTT,
THOMAS REED,
JONATHAN PEARSON,
M. N. LAMOVEN,
HIRAM H. PENNY,
HORACE POTTER.

Resolutions offered at Dartmouth College.

Resolutions adopted at the close of the Lectures upon History, delivered by the Rev. John Lord, in the Chapel of Dartmouth College, July, 1849.

Resolved, That, in this country, the most interested and yet the least read of all countries in the knowledge of the past, *History* deserves a more important place in Education than it has anywhere received among us.

Resolved, That, although it must be admitted to be at least doubtful whether we do not still owe our best National History to a Foreigner, it is encouraging to American genius, and grateful to our patriotism to know that American mind has investigated the Spanish Occidental History, and is now exploring the obscure but fruitful and marvellous records of Mediæval Europe with singular and brilliant success.

Resolved, That the audience tender their sincere thanks to Mr. Lord, for his able and eloquent Lectures, so fitted by the happy choice of his subjects, and by vitality and freedom of thought, both to awaken and direct inquiry upon the least understood, and to us, perhaps, most valuable portions of European history.

The foregoing Resolutions, offered by Professor Haddock, and seconded by Professor Sandborn, were unanimously adopted.

CHARLES B. HADDOCK.

From the Citizens of Philadelphia.

Philadelphia, *January* 2, 1851.

Rev. John Lord,—Sir,—The distinguished success with which you have lectured on History, both in Europe and your own country, naturally excites, among the citizens of Philadelphia, a desire to share in the pleasure and instruction which you are able to afford. Six Lectures on the *Saints and Heroes of the Middle Ages,* which you have delivered in other cities, have been represented to us as striking specimens of the manner in which you discuss historical subjects; and it gives us pleasure to express the opinion that the opportunity of listening to those Lectures, or to any similar course from you, would be cordially welcomed by many in this community.

Very respectfully, your obedient servants,

ALONZO POTTER,
WM. BACON STEVENS,
J. R. INGERSOLL,
HENRY REED,
THO. SERGEANT,
H. M. PATTERSON,
SAMUEL H. PERKINS,
And twenty-four others.

Of the book itself some estimate may be formed from the following testimonials, selected from among a great number received. It is used in many Colleges in the country, among them Yale and Dartmouth.

Copy of a Letter from PRESIDENT SPARKS, *of Harvard College.*

CAMBRIDGE, *April* 4, 1850.

DEAR SIR,—Allow me to thank you for the copy of your Modern History, which you had the goodness to send me. With such leisure as I could command I have perused several of the chapters. The work appears to me well adapted to attain the objects you propose. Much skill is shown in the selection of facts, and in preserving the natural order of events. The narrative is clear, the style is animated and perspicuous, the estimate of the characters and motives of the prominent actors is discriminating and judicious, and above all, there is an enlarged and generous spirit running through the whole which produces the conviction that the author everywhere aims at truth, impartiality and strict justice.

With best wishes for the success of your efforts to render attractive and useful the great lessons of history,

I am respectfully and truly yours,

JARED SPARKS.

Copy of a Letter from REV. DR. TAPPAN, *formerly Professor of History and Philosophy in the University of New York.*

NEW YORK, *April* 5, 1851.

I have used Mr. Lord's History in my school, and am satisfied from the experiment, that it is one of the most valuable text-books in this department which has been issued from the press in this country or in England. It unites the qualities of brevity and clearness, with a power to interest which is rarely found in works of this class.

Copy of a Letter from PROF. H. WEBSTER, *Principal of the New York Free Academy.*

NEW YORK, *February* 22, 1850.

I have read with pleasure the Modern History, from the time of Luthe. to the fall of Napoleon, by the Rev. J. Lord. This book possesses great merit; it is written in a most captivating style, with a strong exhibition of the love of truth. I scarcely know any work on history as interesting, or better calculated to answer a most valuable purpose in the cause of education; besides being especially useful as a text-book, it may be read to advantage by almost any person in any walk of life.

Extract from an article in the BIBLICAL REPOSITORY *and* PRINCETON REVIEW, *April*, 1850.

It was one object of Mr. Lord to furnish, in somewhat moderate compass, for the use of students and young persons generally, a substitute for those wretched, lifeless skeletons, with which publishers and paid book makers are flooding us, under the name of abridgements, or histories for the use of schools. We have lately had the opportunity of hearing the young members of a family, enjoying the advantage of the most popular schools in an adjoining city, preparing their recitations on history. The process was just such as might have been anticipated from the character of the books they were "studying." The sentences were cut up into clauses, containing half an idea, or no idea at all, and sometimes even the most palpable falsehood, and all memorized, with the same unthinking, parrot-like repetitions; until by the law of physical association, the utterance of the word drew after it the utterance of the next, and so the sentence and paragraph were finally completed. Of course history cannot be learned in any such way; nor indeed any thing else of the kind, (for we found children trying to learn astronomy, natural philosophy, chemistry, natural history, and we know not how much more, in the same way;) and for the purposes of education, the effect of the process seemed to us to solve the problem of developing the minimum of intellect, and supplying the minimum of useful and wholesome knowledge. We do not believe any one could practise this method upon the volume before us. It has too much vitality, to be cut up into inch pieces, for the purpose of study. The most conspicuous characteristic of Mr. Lord as a historian, is enthusiasm in his favourite subject. And like all genuine enthusiasm, it imparts itself to his reader.

The book before us goes over the most important period in the history of the world—a period of three hundred eventful years! The author lays claim to no originality of investigation: how could he, in a historical compend of three centuries? but the arrangement, the style, and the sentiments are his own.

Our literary institutions have long felt the need of such a work. Tytler, full of dry statistics, with no beauty of style or diction, is unfit for a text-book. Taylor's Manual, although well-written, lacks method, and perplexes rather than instructs the mind.

Mr. Lord has wove into his work, all the leading features and events of that long and important age, and clothed the whole in his own happy and agreeable style of thoughts. He merits the scholar's thanks for so instructive a book, and we hope it may meet with an extensive circulation.—*New York Protestant Churchman.*

Copy of a Letter from PROF. WEST, *Principal of Rutgers Institute, New York.*

Mr. Lord's work on Modern History is one of the most valuable contributions to school literature that has been made in many years. It is not composed of shreds and patches, as are most of the treatises in this department of knowledge, but elaborated from a mind *imbued* with the spirit of history. It is a living book, and presents the great events of an age in an attractive manner. Its style is beautifully simple and graphic. It is remarkable for its condensation and clearness, and is eminently free from narrow and sectarian views.

I know of no book of the kind so well fitted for the purpose of education as this. It has been used in this Institution the past year, and so great has been the interest taken in it by my pupils, that I feel warranted in recommending it to the attention of teachers generally.

CHAS. E. WEST.

Rutgers Institute, New York,
April 7, 1851.

The best recommendation which can be given to Lord's History is that it recognizes a God in History, and assigns Him His proper agency in the government of this world.—*Christian Secretary, Hartford, Ct.*

It is rather late in the day to produce a "Modern History" which can lay claim to much of originality or research, but Mr. Lord's aim is not to compete on these grounds with his predecessors on the same field, but to simplify and concentrate, according to his own system of arrangement, the facts and data which go to make up the sum and substance of the many histories already before the world. The work is intended for the use of schools and instructors, and in accordance with this plan its method has been adopted.—*New York Literary World.*

This is a volume of very attractive appearance, prepared by a well-read and warm-hearted man, full of his subject, full of matter, and full of scholar-like enthusiasm. It goes over the ages from the time of Luther to the fall of Napoleon. Though professedly written "for the use of schools and colleges," it is admirably fitted for the instruction of that best of all schools—the domestic circle.—*Boston Puritan and Recorder.*

Lord's History contains a vast amount of valuable information on subjects of which no one should be ignorant. It betrays no spirit of political prejudice or religious bigotry; and for the most part derives its materials from unexceptionable sources. Its general correctness cannot be impeached.—*Southern Christian Advocate, Charleston, S. C.*

PINNOCK'S HISTORICAL SERIES.

PINNOCK'S ENGLAND.

REVISED EDITION.

PINNOCK'S IMPROVED EDITION OF DR. GOLDSMITH'S HISTORY OF ENGLAND,

FROM THE INVASION OF JULIUS CÆSAR

TO THE DEATH OF GEORGE THE II.

WITH A CONTINUATION TO THE YEAR 1845:

WITH QUESTIONS FOR EXAMINATION AT THE END OF EACH SECTION:

BESIDES A VARIETY OF VALUABLE INFORMATION ADDED THROUGHOUT THE WORK,

Consisting of Tables of Contemporary Sovereigns and eminent Persons, copious Explanatory Notes, Remarks on the Politics, Manners and Literature of the Age, and an Outline of the Constitution.

ILLUSTRATED WITH NUMEROUS ENGRAVINGS.

FORTY-FIFTH AMERICAN, CORRECTED AND REVISED FROM THE THIRTY-FIFTH ENGLISH EDITION.

BY W. C. TAYLOR, LL. D., OF TRINITY COLLEGE, DUBLIN,
Author of a Manual of Ancient and Modern History, &c. &c.

HISTORICAL SERIES.

PINNOCK'S FRANCE,

HISTORY OF FRANCE AND NORMANDY, FROM THE EARLIEST TIMES TO THE REVOLUTION OF 1848,

WITH QUESTIONS FOR EXAMINATION AT THE END OF EACH SECTION.

BY W. C. TAYLOR, LL. D., OF TRINITY COLLEGE, DUBLIN,

Author of a Manual of Ancient and Modern History, &c. &c., and Editor of Pinnock's Improved editions of Goldsmith's Greece, Rome, and England.

ILLUSTRATED WITH NUMEROUS ENGRAVINGS.

FIRST AMERICAN FROM THE THIRD ENGLISH EDITION.

PINNOCK'S ROME,

REVISED EDITION.

PINNOCK'S IMPROVED EDITION OF DR. GOLDSMITH'S HISTORY OF ROME,

TO WHICH IS PREFIXED

AN INTRODUCTION TO THE STUDY OF ROMAN HISTORY,

AND A GREAT VARIETY OF INFORMATION THROUGHOUT THE WORK,

ON THE MANNERS, INSTITUTIONS, AND ANTIQUITIES OF THE ROMANS;

WITH QUESTIONS FOR EXAMINATION AT THE END OF EACH SECTION.

TWENTY-FIFTH AMERICAN, FROM THE NINETEENTH LONDON EDITION, IMPROVED

BY W. C. TAYLOR, LL.D.,

WITH NUMEROUS ENGRAVINGS BY ATHERTON AND OTHERS.

PINNOCK'S GREECE,

REVISED EDITION.

PINNOCK'S IMPROVED EDITION OF DR. GOLDSMITH'S HISTORY OF GREECE,

REVISED, CORRECTED, AND VERY CONSIDERABLY ENLARGED,

BY THE ADDITION OF SEVERAL NEW CHAPTERS, AND NUMEROUS USEFUL NOTES.

WITH QUESTIONS FOR EXAMINATION AT THE END OF EACH SECTION.

TWENTY-FIFTH AMERICAN, FROM THE NINETEENTH LONDON EDITION, IMPROVED

BY W. C. TAYLOR, LL.D.,

WITH NUMEROUS ENGRAVINGS, BY ATHERTON AND OTHERS.

Pinnock's England, Greece, Rome, and France, have become school classics. In order to make this series more complete, the volumes have been revised by that well-known historian, W. C. Taylor, LL. D., of Trinity College, Dublin.

The popularity of these books is almost without a parallel. Teachers unacquainted with them, will on examination give them a decided preference to any other historical series published.

From the Pennsylvania Inquirer, Philadelphia.

PINNOCK'S GOLDSMITH'S GREECE, ROME, AND ENGLAND.—The popularity of these histories is almost without a parallel among our school books. Their use is co-extensive with the English language, and their names are familiar to all who have received an English education. But if permitted to remain as they came from the hands of the author, they would soon be antiquated; for not only is the stream of modern history flowing onward, but numerous scholars are constantly making researches into that of ancient times. These works are therefore frequently revised, and thus the labours of successive individuals are added to those of the gifted man who wrote them. The present edition is quite an improvement on the former ones. Several important matters which had before been omitted, have been introduced into the text, numerous notes and several new cuts have been added, and every chapter commences with one or more well selected poetical lines, which express the subject of the chapter, and will assist the memory as well as improve the taste of the student. We feel assured that these additions will increase the reputation which these works have hitherto so deservedly sustained.

From JOHN M. KEAGY, *Friends' Academy, Philadelphia.*

I consider Pinnock's edition of Goldsmith's History of England as the best edition of that work which has as yet been published for the use of schools. The tables of contemporary sovereigns and eminent persons, at the end of each chapter, afford the means of many useful remarks and comparisons with the history of other nations. With these views, I cheerfully recommend it as a book well adapted to school purposes.

From MR. J. F. GOULD, *Teacher, Baltimore.*

Having examined Pinnock's improved edition of Dr. Goldsmith's History of Rome, I unhesitatingly say, that the style and elegance of the language, the arrangement of the chapters, and the questions for examination, render it, in my estimation, a most valuable school book:—I therefore most cheerfully recommend it to teachers. and do confidently rust that it will find an extensive introduction into the schools of our country.

HISTORICAL SERIES.

From the New York Evening Post.

A well written and authentic History of France possesses unusual interest at the present time. It becomes especially valuable when, as in the present case, it has been prepared with questions as a text-book for common schools and seminaries, by a scholar so accomplished as Dr. Taylor The work has passed through three editions in England. The American editor has added one chapter on the late revolutions, bringing the history down to 1848, and has added to its value by illustrations throughout, portraying the costume and the principal events of the reigns of which it treats.

This treatise goes back to the origin of the Celtic race, or the Cimbrians, as the offspring of Gomer, peopling the north and east of Europe on the one hand, and to the descendants of Cush—under the names of Scythians, Tartars, Goths, and Scots, warlike, wandering tribes, on the other, tracing the migrations of the latter till they drove the Celts westward, and the Rhine forms the boundary between the two nations. From the Gauls it goes on to the reign of the Franks, Charlemagne, the Carlovingian race, the history of Normandy, and the history of France from the first crusade through its lines of monarchies and its revolutions, to 1848. The style is clear and forcible, and from the compactness of the work, forming, as it does, a complete chain of events in a most important part of the history of Europe, it will be found interesting and valuable for general readers, or as a text-book in our schools. It is comprised in 444 pages, 12mo., and contains a chronological index and genealogy of the kings of France.

Want of space prevents us from inserting all the recommendations received: we however present the names of the following gentlemen, who have given their recommendations to the Histories:

SIMEON HART, Jr., *Farmington, Conn.*
REV. D. R. AUSTIN, *Principal of Monmouth Academy, Monson, Mass.*
T. L. WRIGHT, A.M., *Prin. E. Hartford Classical and English School.*
REV. N. W. FISKE, A.M., *Professor Amherst College, Mass.*
E. S. SNELL, A.M., *Professor Amherst College, Mass.*
REV. S. NORTH, *Professor Languages, Hamilton College, N. Y.*
W. H. SCRAM, A.M., *Prin. Classical and English Academy, Troy, N. Y.*
JAMES F. GOULD, *Principal of Classical School, Baltimore.*
A. B. MYERS, *Principal of Whitehall, Academy, New York.*
HORACE WEBSTER, *Professor Geneva College, N. Y.*
W. C. FOWLER, *Professor Middlebury College, Vermont.*
B. S. NOBLE, *Bridgeport, Conn.*
REV. S. B. HOWE, *Late President of Dickenson College.*
B. F. JOSLIN, *Professor Union College, N. Y.*